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A Survey of Valuation issues for Hybrid Retirement Income Scheme products

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A survey of valuation issues for hybrid retirement income scheme products

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Abstract

So-called variable annuity products which provide certain guaranteed minimum benefits are popular in the United States and Europe but are only in their infancy in Australia. We consider hybrid products with minimum withdrawal rates guaranteed for a fixed term or for life and the financial derivatives embedded in them. A capital markets style model can be used to value the product issuer’s liability under the contract and measure their exposure to equity prices, interest rates, volatility and mortality risk. We survey recent literature for analysis of the valuation challenge faced by product issuers. We find that the guarantees generate material exposure for the issuers and that model based valuation can be informative for a dynamic hedging strategy or to derive risk sensitive measures of required capital and reserves.

1. Introduction

Australians are likely to benefit from increased choice in the range of products available to manage their risks in retirement (Australian Treasury, 2014). These risks include longevity risk – the risk of outliving one’s income stream. Australian retirees have embraced account-based pensions since they provide some flexibility with regard to choice of investment risk, withdrawal schedule, access to capital and which also leave open the possibility of making a bequest. To support development of the product range the Government must consider the direct trade-off between the ongoing public costs of retirement income tax concessions and the potential future public benefits of a reduced burden on the Aged Pension. There are presently available a small number of so called hybrid-products available in Australia (and many in some other countries) which are in effect financial derivatives which provide some
level of income or capital protection within a long term investment for retirement. Such products may include longevity insurance if benefits are guaranteed for life. It is not always possible for these products to provide features consistent with the flexibility demanded by consumers and yet to comply with regulations which would allow the underlying assets to attract the tax concessions afforded to complying retirement income streams. Pricing and risk management challenges are also significant for potential issuers of hybrids which has an impact on the supply of products to consumers.

2. Hybrid products

Hybrid products combine features of account-based products and annuity products. The products are usually account-based, with the beneficiary bearing the investment risk on the underlying assets, subject to certain minimum benefits guaranteed by the product issuer. These so-called variable annuities have been popular in North America and Europe for decades but are only in their infancy in Australia. Customers can choose from various riders to add guaranteed benefits to their annuity contract to create, for example, a guaranteed minimum death benefit (GMDB), a guaranteed minimum income benefit (GMIB) or a guaranteed minimum withdrawal benefit (GMWB). The acronym GMxB is sometimes used for this family of products, where ‘x’ indicates the type of benefit.

We can further differentiate the products by whether they protect benefits for a fixed term or for life. A popular choice of guarantee rider in foreign markets has been the so-called ‘GMWB for life’, sometimes abbreviated to GLWB which stands for ‘guaranteed lifetime withdrawal benefit’. This is an account-based product where the policyholder can make flexible withdrawals within a defined range to provide a retirement income stream. If the account balance is reduced to zero prior to the death of the policyholder, due to the combined effect of low investment returns and compliant withdrawals, then the product issuer is obliged to continue making defined minimum income payments for the remaining life of the insured (see Figure 1). Product terms may allow for flexible withdrawals outside the defined range but this may entitle the product issuer to make corresponding adjustments to the level of guaranteed benefits.

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1 According to Steinorth and Mitchell (2015) more than 80% of variable annuities in place in 2015 and more than 60% of new sales include the GLWB rider.
Scenario a: A 65 year old person takes out a policy with a $100 base value and a withdrawal rate of 5% of the base amount guaranteed for their lifetime. For the first 5 years they withdraw $5 p.a. Following a period of poor returns on investments they reduce their withdrawal to $4 p.a. for the next 5 years b. At age 75 they resume withdrawing $5 p.a. and continue to do so until the account value is exhausted at age 85. Guaranteed payments of $5 p.a. continue until their death at age 95.

Notes: (a) For ease of illustration payments are presented as though they occur annually. The product has no ratchet feature. The guaranteed income and fee rates are 5% and 1% respectively of the guarantee base amount ($100). The account value reflects the investment return on the account, reduced by the guarantee fee and investment management fees and by withdrawals. In the artificial scenario presented above the arithmetic average annual rate of return on the underlying assets after investment management fees but before withdrawals and guarantee fees is approximately +2%. (b) We assume that the specified dollar withdrawal amounts satisfy age-based minimum pension payments prescribed in superannuation rules.
In the remainder of this report we will focus on GMWB and GLWB product types since they can provide flexible account-based solutions which are consistent with previously observed preferences of Australian retirees and which can assist in the management of longevity risk.

Product issuers

In principle, hybrid products can be created by insurers or other financial service firms such as banks. An insurance company with experience offering term and life annuities may acquire the expertise to include guarantee features in their products. Banks with experience managing derivatives and investments may be able to extend their product range to include lifetime guarantees if they can find appropriate means to manage their mortality risk (including by reinsurance). Since banks and insurers are subject to different types of prudential regulation they are likely to hold different perspectives on the valuation and risk management of any hybrid products they issue.

In the following sections where we discuss theoretical valuation and risk management issues we will draw a distinction between insurers and bank issuers where relevant. These descriptors are used loosely to convey the notion that insurers tend to have resources, expertise and a regulatory framework more aligned to the provision of life insurance than for derivative risk management, whilst the opposite may be true for banks. We do not hold that the description is accurate for any particular financial service firm offering such products.

3. Derivative exposures embedded in hybrid products

Financial derivative products tend not to be viewed favourably in the context of retail products particularly after the leverage-induced global financial crisis. Account-based products and guaranteed products may both eschew the use of derivatives in the portfolio of assets. However the contingent nature of the guaranteed benefits of GMWB’s and GLWB’s embody the very essence of derivatives. The policyholder bears all the investment risk while the account has positive value but the guarantor has a ‘ruin-contingent’ obligation to provide

2 The corporate structure of large financial service firms will typically include numerous entities some of which will be appropriately licensed to provide banking, insurance, derivatives or investment management services.
specified income payments should the account value fall to zero, for a fixed term or for the remaining life of the insured.

A GMWB is sometimes described as a variable annuity with an embedded put option (the holder will automatically put the account assets to the issuer in exchange for an annuity, contingent on the ruin of the assets). Alternatively it can be described as a ruin contingent call on the annuity. Whether we describe the option as a put or a call is moot. What is clear is that the option generates a complex hybrid of equity and interest rate risk. The likelihood of ruin of the account will be primarily a function of equity\(^3\) prices, equity volatility and interest rates (since interest rates affect forward equity prices). The value of the annuity assets to be exchanged for the account asset (on ruin) will be a function of the term structure of interest rates from the likely time of ruin.

Most of the capital market exposures of the GLWB will be similar in nature to the exposures generated by the GMWB since the basic characterisation of the cash flows of each product are the same, except that the term of the GLWB annuity that will arise on the ruin of the account will be the remaining lifetime of the person insured.

The product issuer is exposed to greater longevity of policyholders after the ruin of the account, but the degree of longevity exposure prior to ruin cannot easily be estimated since it will depend on a complex interaction between equity, interest rates and the age of the policyholder. For example, longevity exposure will be high for accounts which are already near to or in ruin, especially if prevailing annuity rates are then lower than the guaranteed rate. By contrast, longevity exposure will be low for accounts which have experienced high rates of return and have low probability of ruin prior to the death of the insured. There will be no liability at all in respect of policyholders who die before their account is ruined.

Possible variation in the composition of the underlying assets in the account to which the option relates is a major departure from the traditional concept of a financial derivative. Such variations may be possible due to flexibility offered to investors who may elect to vary their asset exposure by choosing from a defined menu of funds (for example a menu may include ‘conservative’, ‘balanced’ and ‘aggressive’ alternatives\(^4\)). Variations may also arise if the

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\(^3\) Typically the ‘risky’ part of the account assets will be comprised of equities.

\(^4\) The choice of fund may also affect the level of ongoing fees paid by the customer.
product issuer has specified contract terms which allow it to vary unilaterally the proportional allocations between high risk and low risk funds in the account in certain circumstances. So-called ‘volatility management’ features in a long term derivative product make it possible for issuers to reduce overall charges for providing a guarantee, and also make it possible to include certain investments on the menu which it would otherwise be too risky for the issuer.

Much of the risk for the issuer in setting the price of a long-term derivative contract arises from estimating the value of certain parameters which cannot be hedged in a market even if they were willing to do so. For example a product issuer would be very unlikely to find a hedge (lasting for two decades or more) for the equity volatility risk inherent in the funds on their product menu. So indirect control of the risk via methods of volatility-management are a very important determinant of pricing.

*Premium payments and policy lapsation*

The customer pays for the guarantee via a fee which is typically calculated as an annual percentage of the guarantee base and deducted directly from the account. Prior to 2008 guarantee fees were typically less than 0.50% but, even before the financial crisis, were assessed by Milevsky and Salisbury (2006) as being under-priced for risk\(^5\). More recently average guarantee fees have been estimated at around 1% (Lavine, 2011), but a wide range of fees will be observed due to the myriad of variations in product terms. It is likely that other contract fees and asset management fees will be charged in addition to the guarantee fee and these may easily comprise another 2-3% p.a. If the product issuer is also earning the asset management fee then it is difficult to assess whether the guarantee fee represents fair value in isolation of other revenue and costs.

If we describe these fees as option premiums we note that the premium is both deferred and contingent. There will be some minor interest rate exposure arising from the deferral of the premium. The size of the premium payments is uncertain if the future guarantee base can vary due to a feature such as a ratchet. Premiums are contingent since they will only be paid up until the ruin of the account, or the lapse of the policy, to which we now turn our attention.

\(^5\) As will be explained later most of the under-pricing related to assumptions about lapsation behaviour.
Policyholders may lapse (withdraw all their capital from) a typical policy at any time. There will usually be a surrender penalty if this occurs within a specified time period after inception. Surrender penalties have typically been in the range of 1-5% but may be as high as 10% (Milevsky & Salisbury, 2006, p. 12) and are necessary for the issuer to counterbalance the risk that the deferred option premiums will not be paid if the policyholders elect to lapse the policy. Were it not for the surrender penalty a rational policyholder seeking to maximise their expected wealth may be inclined to lapse a policy soon after inception if the account balance experiences strong (or even modest) growth and if they perceive that the future fees that they will pay are greater than the prevailing value of the guarantee plus the surrender penalty. This product feature generates some risk characteristics similar to American style put options in respect of which it may be optimal for the holder to exercise prior to maturity.

In addition to the option to lapse, policyholders usually have some flexibility with regard to the level of withdrawals that may be made from the account in each annual period. Withdrawals will be subject to a maximum limit (without penalty) since excess withdrawals reduce the account value and make it more likely that the guarantee will be required.

At one extreme, it is possible to value the contract under the assumption that policyholders will act rationally to maximise the expected present value of their contract and in doing so determine an optimal strategy for both withdrawal and lapsation of their policy. A value for the contract can be obtained by adding the discounted expected value of the cash flows arising under such an optimal strategy. This is a sound and conservative approach for the valuation of vanilla American style options in financial markets and as such may appeal to a bank-issuer of a hybrid product. Any sub-optimal behaviour by the option holder would give rise to (usually small) windfall gains to the option writer at the time of sub-optimal exercise or non-exercise.

This approach will probably be too conservative for retail products where consumer behaviour may be quite different to that expected under the assumption of economic rationality. Moreover, retail investors tend to demonstrate high levels of inertia with respect to major financial decisions rather than being tactical and timely. If the bank issuer assumed that all investors would behave like risk-neutral, rational economic agents then their guarantee fees would likely be so high as to make their product uncompetitive in the
marketplace. An insurer may be best placed to make a more competitive price which includes an actuarial assumption of some level of sub-optimal behaviour by policyholders\(^6\).

*Bells and whistles*

A customer may be allowed to select other features (‘bells and whistles’) to add on top of their GMWB or GLWB policy. Some common features\(^7\) are described below.

- **Ratchet**: The guaranteed amount is potentially increased on each policy anniversary to the larger of the current guarantee level and the account value (ratchets are the most common enhancement to a GLWB (Steinorth & Mitchell, 2015, p. 251).  
- **Step-up**: The maximum allowable withdrawal rate is increased towards the end of the term if no withdrawals are made earlier during the term.
- **Roll-up**: A guaranteed compounding percentage increment that will be made to a benefit (such as the guarantee amount or a death benefit) each year.
- **Death benefit**: To protect against early death during the policy an additional premium may be paid to secure the payment of a death benefit to the estate of the policyholder.

We do not intend to comment on the incremental value of such product enhancements. However we can readily anticipate the type of impact that such features may have on the risk profile for the product issuer. For example the ratchet feature can be anticipated to increase exposure to higher equity volatility, since higher volatility increases the likelihood of the guarantee liability being increased. The death benefit creates mortality risk for the insurer but there may be some favourable offset against longevity risk which the product has already created. We can also anticipate that these features may impact the withdrawal strategy of the policyholder and their propensity to lapse, although there is little prospect of making an accurate prediction of the size of the impact on the strategy.

4. **Modelling stochastic processes**

We have recognised that volatility in the levels of equities and interest rates and will affect the value of GMWB contracts, as will volatility in mortality rates in the case of GLWB contracts.

\(^6\) A well known example of suboptimal financial behaviour by U.S. households is the failure of many to exercise an option to refinance their mortgage at lower rates (Keys, Pope, & Pope, 2014).

\(^7\) For a longer description of the array of features available in US products see Bauer, Kling and Russ (2008) and for European products see Bacinello, Millossovich, Olivieri and Pitacco (2011).
It is possible, but not necessarily productive, to construct a complex model with multiple elements modelled as stochastic processes. In the literature it has been more typical to include a small number of stochastic variables which are most relevant to the particular set of product features under examination, whilst other real world variables are treated as deterministic or exogenous parameters.

*Arbitrage-free valuation*

The backbone of the valuation methodology applied to capital market derivatives is risk-neutral or arbitrage-free valuation. It is well known that if one makes an assumption that market participants can construct risk-free portfolios by trading in both the derivative and its underlying asset then the derivative can be valued by taking the discounted expected value of its future cash flows using a risk-free interest rate. In the real world investors may have risk preferences which are not risk-neutral and have expectations about returns which are not the same as the risk free rate. However, the consequence of the ‘no arbitrage’ assumption is that the valuation in the real world should be the same as the valuation in a risk-neutral world (Hull, 2008, p. 241).

Of course no bank actually rebalances its hedges continuously and, in any case, all the other model parameters like volatility which were assumed to be fixed are not actually so. Nevertheless, the no arbitrage framework is the standard paradigm for bank risk management of derivatives (at least to begin with) provided that there is a liquid market in the underlying asset so that dynamic hedge rebalancing can at least be attempted, and provided that a decent option market exists for volatility hedging. It is arguable that the arbitrage-free approach to valuation may not be entirely appropriate for hybrid products with mortality risk since insurance markets are incomplete, so dynamic hedging can only be partially implemented at best (Bacinello et al., 2011, p. 10; Milevsky & Salisbury, 2006, p. 3). We also observe that an insurer issuer of hybrid products may never intend to employ active dynamic hedging, instead preferring a frequentist or actuarial approach to pricing and appropriate reserving.

Perhaps the most important point is that regardless of whether a bank or an insurer actually employs dynamic hedging, the model valuation based on the no arbitrage assumption can provide sensitivity measures which respond immediately to changes in capital market inputs and any other parameters. A key feature of the modern approach to prudential risk
management of capital and reserves is that measures of risk are sensitive to market conditions and to any deterioration in the portfolio of risks taken. This allows capital and reserve levels to move in tandem with actual levels of risk. So model generated sensitivities for hybrid products can still provide useful inputs for calculating risk capital and reserves even when the actual valuation and risk management strategy differs from the model assumptions.

Equity models

Most authors employ a standard model for the evolution of equity prices with deterministic interest rates and equity volatility so that equity prices will exhibit the so-called log-normal distribution associated with the familiar Black-Scholes option pricing model. For ease of analysis the whole of the variable annuity sub-account may be assumed to evolve according to this process (notwithstanding that the account may contain smaller amounts of other assets like cash which are less risky than equities) with an appropriate estimate of the volatility of the total account value. Examples of papers using this equity model include Milevsky and Salisbury (2006), Bauer et al. (2008) and Dai, Kwok and Zong (2008). In some cases there is explicit acknowledgement that equity volatility is assumed a priori to be much more important to the optionality of the contract than the interest rate, since the latter will primarily affect only the discount factors (Dai et al., 2008, p. 597). Kling, Ruez and Russ (2011) employ a more advanced model which also includes stochastic equity volatility. The average level of volatility is found to be more important than the fact that volatility is stochastic although it can have a material impact on hedge ratios.

Interest rate models

Whilst it may be acknowledged generally that equity volatility is more influential than interest rate volatility to these products it has been observed that joint outcomes of equities and interest rates matter to the product issuer. The worst capital market outcome for the issuer is a fall in equity prices which drive the account value to zero whilst interest rates simultaneously fall, making annuities more expensive. To get an explicit measure of instantaneous exposure to the correlation between equities and interest rates a model is needed in which both are stochastic and their processes correlated. Peng, Leung and Kwok (2012) implement a model for analysing pricing behaviour of a GMWB product with a Vasicek model for the short term interest rate and a standard geometric Brownian motion for equities
and the two stochastic processes are correlated. Their results confirm that the value of GMWB put option increases with positive correlation between equities and interest rates. Sensitivity to correlation is lower than sensitivity to some other parameters but likely to be more material than can be safely ignored. The results also indicate that given a conservative setting for the correlation parameter, sensitivity of the fair value guarantee fee to interest rate volatility becomes material.

*Deterministic lapse and dynamic lapse*

The simplest valuation model, which may still be appropriate for examining characteristics which are independent of lapse behaviour, is to specify a deterministic schedule by which a pool of contracts will be depleted each year due to withdrawals and lapses based on prior experience of consumer behaviour in related products. Milevsky and Salisbury (2006) model a deterministic withdrawal strategy for a GMWB and use that valuation as a baseline with which to compare valuations derived assuming that policyholders are fully rational and seek to optimise the value of their option by lapsing the contract when it is optimal to do so. They characterise the contingent outcome as an optimal stopping problem and use the Monte Carlo least squares method developed by Longstaff and Schwartz (2001) to value a component of the product as an American style option. Bauer et al. (2008) take the approach suggested by Milevsky and Salisbury and apply it to a wider array of GMxB type products.

Dai, Kwok and Zong (2008) derive valuations by solving an optimal control problem for the optimum withdrawal strategy that should be followed by a risk neutral rational policyholder who is allowed to withdraw more than the contracted withdrawal limits subject to the payment of penalty fees. Bacinello et al. (2011) explore three possible withdrawal strategies which they term static, dynamic and mixed. Static means a fixed withdrawal schedule without lapses. In the dynamic model there is a stochastic withdrawal and lapse which generate scenarios from which the worst case is selected to derive the ‘least optimal’ valuation for the insurer. In the mixed strategy the policyholders are assumed to follow a static withdrawal path but the model assumes a stochastic lapse strategy.

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8 We graphically interpret results presented in Peng et al. (2012) to indicate that the total value of the put option varies by approximately 0.5% of the guarantee amount for a 0.1 variation in the correlation coefficient, given reasonable values for other parameters.
Finally, recognising that policyholders are very unlikely to have the means or inclination to attempt to determine their withdrawal or lapse strategy by optimising their expected present value, some issuers may be inclined to use a plausible heuristic strategy as a model of policyholder behaviour, such as a simple rule of thumb for estimated lapse rates based on the ratio of the account value to the guarantee value, which may be taken to indicate the degree to which the embedded option is in or out-of-the-money (Ngai & Sherris, 2011, p. 7).

Flexible choice of funds and volatility management

Allowing policyholders to have flexibility with regard to the choice of fund(s) in their sub-account, or more generally to allow them to determine the proportional allocation to ‘risky’ investments, is a major potential source of uncertainty with regard to the long term average volatility that will be experienced by the account. Volatility management can generate a material reduction in modelling complexity. Product issuers are likely to show a strong preference for volatility management controls, and other softer controls on the menu of risk options offered to consumers, rather than modelling stochastic volatility.

Mortality models

Capital market models can be extended to incorporate stochastic mortality (Bacinello et al., 2011; Ngai & Sherris, 2011) but GLWB products tend to generate relatively little longevity exposure which arises only in the most adverse of markets. Insurance markets are not liquid enough to allow active rebalancing of a hedge against longevity risk so simpler models with deterministic mortality assumptions will be preferred in practice. Further, mortality risk will almost certainly be dominated by the assumptions made about lapse rates.

GMWB and GLWB valuation

Milevsky and Salisbury (2006) found a range of 0.73% to 1.60% p.a. (of assets) for the fair value of the annual guarantee fee for a 14 year GMWB with a 7% withdrawal rate and a 1% surrender penalty. The authors reported prevailing market prices of 0.3%-0.5% for similar products and concluded that the market was substantially under-pricing the risk, despite the possibility of sub-optimal lapsation behaviour by policyholders. Bauer et al. (2008) find a fair value guarantee rate of 0.19% p.a. for a similarly specified GMWB except with a much higher

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9 Prescient, given that the paper was published prior to the global financial crisis.
surrender fee of 5% and with deterministic lapse. The fair value fee more than doubled if they assumed rational lapse behaviour. Dai et al. (2008) report a valuation range of 0.73%-1.65% p.a. with a 5% surrender penalty. The upper end of the range reflects pricing under the optimal controlled withdrawal strategy.

Fung et al. (2014) focus on the impact of mortality assumptions on valuation. They only consider a model with a static withdrawal strategy. They find the fair value for the annual guarantee fee to be about 0.5% p.a. using base case assumptions for the market price of mortality risk and the volatility of mortality. They demonstrate that valuation sensitivity to the mortality parameters is non-negligible, but much lower than sensitivity to equity volatility.

Balancing product features against valuation

Product issuers can determine a trade off between product features and profit margin to achieve headline guarantee fee rates that they believe will be appealing to consumers. For example, when market interest rates are low (so expectations of annuity rates are low) issuers may reduce the guaranteed withdrawal rate for new offerings to keep the fees down. Similarly, issuers must choose an appropriate balance between the surrender penalty fee and the annual fee based on and their expectations of policyholder lapsation behaviour. Finally the issuer can seek to mitigate equity volatility risk with (potentially) softer looking volatility management controls rather than by simply building a more conservative profit margin into the annual fee.

Relationship between modelling and hedging

It may be natural for a bank issuer of hybrid products to seek to hedge variation in the profits or losses arising from an arbitrage-free valuation model such as one of the stochastic modelling approaches described in the previous section. Relevant and permissible hedges in the trading book will reduce the net exposure arising from the product issuance and thereby reduce the VaR\textsuperscript{10} capital requirement for the bank.

\textsuperscript{10} The VaR (“Value at Risk”) model is widely used in the regulation of financial services firms to estimate the loss percentile of a distribution such as profit and loss at a specified level of confidence (Bessis, 2010, pp. 200-217).
An insurance company will need to hold sufficient reserves against the contingent liabilities created by the product and sufficient capital to ensure its ongoing solvency. An actuarial approach to assessing the worst case loss at a defined probability may not be consistent with a stochastic modelling approach. Actuarial parameter estimates and valuations will be conservative but not likely to be updated as frequently as would be the case using a capital markets model. As is the case for banks, there is a desire for the capital requirements in the insurance industry to be risk-based (Milevsky & Salisbury, 2006, p. 3). So even if the insurance company issuer of a hybrid product is not proposing to dynamically hedge the exposures in the same way as a bank, a model based valuation is still essential to allow the calculation of risk-based capital requirements.

5. Conclusion

An issuer of GWMB or GLWB products is exposed to risks similar to those experienced by any issuer of a long term derivative contract, in this case with exposure to the levels of and volatility of equity prices, interest rates and mortality rates. A valuation model built using components which are familiar in the context of capital markets can be useful in assessing the current valuation of the GWMB and GLWB contracts issued, and their sensitivity to changes in market prices and parameter assumptions. The derivative valuation model approach is informative for determining hedging strategies and for deriving measures of required capital and reserves which are appropriately sensitive to risk.

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References


