Connecting the dots: econometric methods for uncovering networks with an application to the Australian Financial Institutions

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Connecting the dots: econometric methods for uncovering networks with an application to the Australian financial institutions

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Abstract. This paper connects variance-covariance estimation methods, Gaussian graphical models, and the growing literature on economic and financial networks. We construct the network using the concept of partial correlations which captures direct linear dependence between any two entities, conditional on dependence between all other entities. We relate the centrality measures of this network to shock propagation. The methodology is applied to construct the perceived network of the publicly traded Australian banks and their connections to the domestic financial sector, real economy, and international markets. We find strong links between the big four Australian banks, the financial services sector and the other sectors of the economy and determine which entities play a central role in transmitting and absorbing the shocks.

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1 Introduction

The global financial crisis has called for better understanding of financial market vulnerabilities and risks. Connections between different segments of markets play an important role in determining the extent and patterns of these risks. These interconnections can be studied using the tools of network theory. Network modeling is a novel and rapidly developing field in social sciences, economics and finance, see e.g., Jackson (2008) and Allen and Babus (2009).

This paper brings together ideas from network theory, the financial econometric literature on variance-covariance modeling, and statistical literature on Gaussian graphical models (GGM). The GGM are widely used for the reconstruction of networks when the actual network structure is unobservable. A prominent example of this use is the biological literature on networks of genes, proteins, etc., see, e.g., Rice et al. (2005). These methods are relatively new for economics and finance.¹ We apply the GGM to reconstruct the network of partial correlations between different Australian banks, domestic economic sectors and international markets. We use network theory to study this network and interpret its properties.

GGM are developed to visualize the conditional dependences between different elements of a multivariate random variable by a graph of partial correlations, see Whittaker (2009) for detailed treatment. Partial correlations capture bi-variate linear dependence between any two elements of the random variable, conditional on a set of all remaining elements. As we show, this feature is useful to separate a direct dependence, between a pair of economic sectors or entities, from indirect effects coming through the remaining part of the network. The standard GGM literature focuses on the reduction of complexity of the conditioning set from the constructed graph (so-called Markov properties). Our primary focus is on the network-based measures used in the theoretical network literature in the context of the graph of partial correlations.

Recent economic literature provides many examples of how financial data can be described from the network perspective.² The studies illustrate the complexity of relationships between financial entities and discover certain network properties which may be

¹See Barigozzi and Brownlees (2013) for another example.
²See Iori et al. (2008) who study the interbank overnight market, Vitali et al. (2011) analyzes the ownership of transnational corporations, and Sokolov et al. (2012) investigates the Australian interbank transactions, among many others.
important for the aggregate properties of the financial system. An intuitive, but not entirely formalized idea is that an edge between nodes represent a channel of transmission of a shock. Thus, the network approach is useful for studying systemic risk, see review of early contributions in Chinazzi and Fagiolo (2013) and recent studies of Acemoglu et al. (2015), Glasserman and Young (2015) and Elliott et al. (2014). Battiston et al. (2012) introduce the DebtRank which is an example of centrality measure of nodes within the network of financial entities. High centrality of a node would reflect an importance of the node in the shock transmission.

This paper is closely related to the recent work of Billio et al. (2012), Dungey et al. (2013), Barigozzi and Brownlees (2013) and Diebold and Yilmaz (2014). The key differences is that we are establishing the links between the statistical concepts of correlations, partial correlations, principal components and various centrality measures from the network theory. Moreover, this is the first work mapping the network of perceived financial dependencies between the Australian banks, other domestic sectors, and international markets. We use publicly available information on the share prices and indices of the corresponding entities to reconstruct the network of partial correlations between their returns. The returns generally represent market perceived changes in the value of these entities. The reconstructed networks may be a useful tool for better understanding of the market, dynamic spreading of the shocks and, hence, may be used for policy and regulatory analysis.

We find that there are strong direct links between the big four Australian banks, which are connected to the real economy, real estate and financial groups. The Australian market is strongly connected to the Asian market.

The paper is organized as follows. Section 2 defines the network of partial correlations. Section 3 discusses key network measures and their relationships to well-established statistical concepts. Section 4 details the estimation procedure. Section 5 applies the network methods to uncover a perceived network of the Australian banks including connections to local financial and real sectors and global markets, and demonstrate relevant policy examples. Section 6 concludes the paper.
2 Networks of partial correlations

Formally, a graph $G = (\mathcal{V}, \mathcal{E})$ is a structure consisting of a set of nodes, $\mathcal{V}$, and a set of edges, $\mathcal{E}$. Every two nodes may or may not be connected by an edge, edges may be directed or undirected, and weighted or unweighted. Our focus here is on undirected graphs and, hence, elements of an edge set $\mathcal{E}$ are unordered pairs $(i, j)$ of distinct nodes $i, j \in \mathcal{V}$. We are working with weighted graphs when each edge has a non-zero weight $w_{ij}$ assigned to it.

Following the literature on Gaussian graphical models (Whittaker, 2009) we define the network of partial correlations. Let $X$ denote an $n$-dimensional multivariate random variable and the nodes of the graph $G$ correspond to each component of $X$, i.e., $\mathcal{V} = \{X_1, X_2, \ldots, X_n\}$. Let $X^*_{i|\mathcal{V}\setminus\{X_i, X_j\}}$ denote the best linear approximation of variable $X_i$ based on all the variables except for $X_i$ and $X_j$ for any pair $i, j$.

**Definition.** Partial correlation coefficient between $X_i$ and $X_j$, $\rho_{X_i,X_j \mid \mathcal{V}\setminus\{X_i, X_j\}}$, is defined as the ordinary correlation coefficient between $X_i - X^*_{i|\mathcal{V}\setminus\{X_i, X_j\}}$ and $X_j - X^*_{j|\mathcal{V}\setminus\{X_i, X_j\}}$.

In other words, the partial correlation between $X_i$ and $X_j$ is equal to the correlation between the residuals of the two linear regressions: (1) $X_i$ on a constant and a set of control variables, which includes all variables in $X$ except for $X_i$ and $X_j$ and (2) $X_j$ on a constant and the same set of control variables as in the first regression. Hence, the partial correlation measures linear dependence between any two components of $X$, $X_i$ and $X_j$ (for $i \neq j$) after controlling for linear dependence with all other remaining components in $\mathcal{V}\setminus\{X_i, X_j\}$. For brevity, we will use a shorter notation $\rho_{ij}$ for the partial correlation between $X_i$ and $X_j$.

The edges of the network of partial correlations correspond to the pairs of random variables with non-zero partial correlations, $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V} \mid \rho_{ij} \neq 0\}$ and the edge weights to the corresponding partial correlations, $w_{ij} = \rho_{ij}$. Intuitively, the network of partial correlations visualizes linear dependence between any two random variables conditional on all other variables. When the random variable $X$ is multivariate normal, zero partial correlation implies conditional independence of the corresponding components. This statement holds for a more general case of an arbitrary continuous marginal distribution for each component of $X$ when the dependence between the components is characterized by the Gaussian copula (see, e.g., Diks et al., 2010).
For better intuition behind the concept of partial correlations it is useful to show connections with linear regression. Project each $X_i$, $1 \leq i \leq n$ on the space spanned by the rest of the variables in $X$ as

$$X_i - \mu_i = \sum_{j \neq i} \beta_{ij} (X_j - \mu_j) + \varepsilon_i,$$ (1)

where $\mu_i$ is the unconditional mean of $X_i$ and $\varepsilon_i$ is a zero-mean residual. Denote variance-covariance matrix of vector of residuals, $\varepsilon$, as $\Sigma$. This matrix is not necessary diagonal as $\varepsilon$’s may be correlated. The diagonal elements of $\Sigma$ are the conditional variances of different components of $X$ after conditioning on all the remaining components, $\text{Var}(\varepsilon_i) = \text{Var}(X_i | \bigvee \{X_i\})$. The orthogonality condition, $E(\varepsilon_i X_j) = 0$, $\forall j \neq i$, $1 \leq i, j \leq n$, implies that regression coefficients are given by $\beta_{ij} = \rho_{ij} \sqrt{\text{Var}(\varepsilon_i) / \text{Var}(\varepsilon_j)}$ (see Appendix A.1).

It follows that $\rho_{ij} = \text{sign}(\beta_{ij}) \sqrt{\beta_{ij} \beta_{ji}}$.

Partial correlations are also related to the (unconditional) variance-covariance matrix of $X$, $\Omega = \text{Cov}(X)$. Define a concentration or precision matrix as the inverse of a non-singular variance-covariance matrix, $K \equiv \Omega^{-1}$. The partial correlations can be expressed as

$$\rho_{ij} = -\frac{k_{ij}}{\sqrt{k_{ii} k_{jj}}},$$ (2)

where $k_{ij}$ is the $(i,j)$ entry of $K$, see Appendix A.2. Furthermore, each diagonal element of $K$ is the reciprocal of a conditional variance, i.e., $k_{ii} = 1 / \text{Var}(\varepsilon_i)$, where $\varepsilon_i$ are defined in Eq. (E).

Finally, the partial correlation can also be computed using the inverse of $R$, the regular correlation matrix of $X$, by replacing $k_{ij}$, $k_{ii}$ and $k_{jj}$ entries in Eq. (2) with the corresponding entries of $R^{-1}$, see Appendix A.3. The $i$th diagonal element of $R^{-1}$ is the ratio of unconditional variance of $X_i$, $\text{Var}(X_i)$, to conditional variance of $X_i$, $\text{Var}(\varepsilon_i)$. The proportion of variation in $X_i$ explained by all the other components, which can be thought as endogenous network-induced variation of component $i$, can be defined similarly to the regression’s coefficient of determination,

$$R_i^2 = 1 - \frac{\text{Var}(\varepsilon_i)}{\text{Var}(X_i)},$$ (3)

and deduced from the diagonal elements of $R^{-1}$. 
A graph with \( n \) nodes can be represented by the adjacency matrix of size \( n \times n \). When the graph is undirected and weighted, as in our case, the adjacency matrix is a symmetric matrix whose \((i, j)\)-entry is not zero only if there is an edge connecting nodes \( i \) and \( j \), and the entry is given by the weight. As we will see in the next Section, adjacency matrix is a basis for other network-based measures.

Let \( P \) denote the adjacency matrix of the graph of partial correlations. The elements of this matrix are \( P_{ij} = \rho_{ij} \) for \( i \neq j \) and zeros on the diagonal. From (2) we have

\[
P = I - D_{K}^{-1/2}KD_{K}^{-1/2},
\]

where \( I \) is the identity matrix of size \( n \), \( K \) is the concentration matrix, and \( D_{K} \) is the diagonal matrix composed of the diagonal elements of \( K \). The diagonal elements of \( K \) are the inverse of the conditional variances of the components of \( X \). Therefore,

\[
D_{K} = \text{diag}\{k_{11}, \ldots, k_{nn}\} = \text{diag}\left\{\frac{1}{\text{Var}(\xi_1)}, \ldots, \frac{1}{\text{Var}(\xi_n)}\right\} = D_{\Sigma}^{-1},
\]

where \( D_{\Sigma} \) is the diagonal matrix composed of the diagonal elements of \( \Sigma \).

Next we show how the adjacency matrix \( P \) relates to system of linear equation (E). Introduce matrix \( B \) with zeros on the diagonal and \( \beta_{ij} \) in the \((i, j)\) off-diagonal entry. Exploring the relationship between \( \rho_{ij} \) and \( \beta_{ij} \) defined above, we find that \( B = D_{\Sigma}^{1/2}PD_{\Sigma}^{-1/2} \).

Then Eq. (E) can be rewritten in a matrix form as follows

\[
X - \mu = B(X - \mu) + \varepsilon = D_{\Sigma}^{1/2}PD_{\Sigma}^{-1/2}(X - \mu) + \varepsilon,
\]

where \( \mu \) and \( \varepsilon \) are the vectors of means of \( X \) and residuals, respectively. While we should be careful about causal interpretation of this equation, it is useful to think about \( \varepsilon \)'s as external shocks influencing the system. Because \( X_i \)'s may have different conditional variances, to compare the effects of the shocks, we rescale the residuals in such a way that all of them would have unit conditional variance, \( e = D_{\Sigma}^{-1/2} \varepsilon \). It is important to emphasize that \( e \)'s are not independent (as well as \( \varepsilon \)'s).

We multiply both sides of the previous equation by \( D_{\Sigma}^{-1/2} \) from the left and introduce
a rescaled variable \( x = D^{-1/2} \Sigma (X - \mu) \). The equation becomes

\[
x = Px + e.
\]  

(5)

Note that the rescaling for \( X \) is in terms of the conditional variance as opposed to a more usual rescaling by the unconditional variance. Intuitively, by using the conditional variance we remove the effect of the variables endogenous to the network.

Using equation (5) we will now demonstrate the usefulness of the network of partial correlations for the systemic risk analysis and its connection with the network of correlations.

### 2.1 Interpretation of Partial Correlation Network

In Section 5 we reconstruct perceived financial networks of the Australian banks, other sectors of the economy and international markets using publicly available information on the returns of the corresponding bank shares and indexes. The returns reflect market perception about the percentage change in the present value of a company, sector or marker overall. By looking at the correlations of the returns for some entities we may uncover how the market perceives joint changes in value of these entities including any intermediate effects. The use of correlations in this sense has been widely used in finance for optimal portfolio selection (Markowitz, 1952). However, if one wants to understand the structure of the market and use it for, say, financial stability analysis or optimal policy design, it is important to turn to partial correlation analysis. Partial correlations can single out the direct co-movements in relative change of values between the pairs of entities controlling for all other entities. By reconstructing the network of partial correlations we are may observe how a unit variance shock may spread through the network. From the perspective of the regulator this allows identifying the most important relations and focusing policy on these relations or mitigating possible consequences of any large shocks to these entities.

It is important to emphasize at this point that with partial correlations, it is not possible to establish the direction of causation. The concept of Granger causality (Granger, 1969) may be used to establish directional relationships, though, the presence of Granger causality would imply predictability and subsequently, may be incorporated by the market. Therefore, unless we turn to high frequencies, it is hardly possible to detect stable and
substantial linear Granger causality in financial returns. Lower frequency cross-sectional
dependencies between the entities are to some degree reflecting aggregations of Granger
causalties at higher frequencies.

With this caution about causality in mind, we propose an observational interpretation
of system (5). The expected steady state value of \( x \) is 0. Suppose that we observe a
deviation in \( x \). We will now decompose the total observed deviation into a direct effect
given by unit-variance shock \( e \) and an indirect (endogenous to the network) effect. Initial
shock \( e_i \) which directly hits node \( X_i \), will also affect the immediate neighbors of \( X_i \). For
the network as a whole, the expected effect of shock \( e \) on the immediate neighbors is
measured by \( Pe \). We call this a first-order effect. The expected effect on the neighbors of
the neighbors, can be computed as \( P^2e \), which we call a second-order effect. Analogously,
we define a \( k \)th-order effect as \( P^k e \). The total effect of the shock on \( x \) will be\(^3\)

\[
e + Pe + P^2e + \cdots = \sum_{k=0}^{\infty} P^k e = (I - P)^{-1} e.
\]

Note that the last expression can be obtained directly from Eq. (5). Using Eq. (4), one
can see that \((I - P)^{-1}\) is equal to

\[
T = D^{-1/2} \Omega D^{-1/2},
\]

so that the total effect of the shock is given by \( Te \). Matrix \( T \) is a variance-covariance matrix
of \( x \), the rescaled \( X \). Eq. (7) shows that it is also the rescaled variance-covariance matrix
\( \Omega \). It looks similar to an ordinary correlation matrix, though, instead of unconditional
variances of \( X \), the conditional variances of \( X \) are used for rescaling.

We have established that \( T \) transforms the initial shock into its total effect on \( X \).
Instead, the matrix of partial correlations \( P \) defines how shocks spillover on the immediate
neighbours.

### 3 Network-based Measures of Centrality

The previous interpretation shows that the network of partial correlations can be used
to separate the direct and higher-order spillover effects of shocks. Understanding and

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\(^3\)The last equality in (6) holds only when all eigenvalues of matrix \( P \) are within the unit circle, which
we will assume to be the case. This assumption is satisfied in our application.
measuring spillovers by means of the partial correlation matrix (as opposed to limiting
attention to the variance-covariance matrix relevant for the total effect of the shock) is
important for a policy aiming to reduce the systemic risk. Some edges of the partial
correlation network (intuitively, those with high weights) and, as a consequence, some
nodes (namely those with many edges with high weights) may play a higher role in these
spillover effects. Network theory uses various measures of centrality to measure a relative
importance of nodes and edges in the graph.

There exist several different centrality measures, and all of them attempt to evaluate
the nodes’ positions in the graph. One of the simplest centrality measures for a graph
is a degree centrality. For a weighted graph, it is defined for each node by adding all
weights of the edges connected to the given node. In our case, for the adjacency matrix
\( P = (P_{i,j})_{i,j=1}^n \) in the partial correlation network, the node’s degree is computed as

\[
d_i = P \cdot 1 = \sum_{j=1}^n P_{i,j} = \sum_{j=1}^n \rho_{ij},
\]

where \( 1 \) is an \( n \times 1 \) vector of ones. Intuitively, the nodes with high degree centrality (in
absolute value) are more important for the transmission of the shock due to having many
edges and/or edges with high weights.\(^4\)

The node whose degree centrality is large will receive more shocks from other nodes
and will also transmit the shocks to the larger number of nodes. But how far can the
shock be transmitted? That obviously depends on how central the nodes directly con-
ected to a central node are. This idea leads to the following self-referential measure of
centrality. Eigenvector centrality is defined simultaneously for all nodes as the eigenvector
\( u_1 \) corresponding to the largest eigenvalue of the adjacency matrix \( P \). It is easy to show
that this eigenvector is proportional to the limit of the iterative application of operator
\( P \) to arbitrary initial vector \( e_0 \in \mathbb{R}^n \). Indeed, symmetric matrix \( P \) has the orthonormal
basis \( \{u_1, \ldots, u_n\} \) with the real eigenvalues \( \lambda_1 \geq \cdots \geq \lambda_n \). Writing \( e_0 \) in this basis with

\(^4\)Note that some entries of \( P \) may have negative value indicating that the sign of the shock will be
reversed for the corresponding nodes. However, in our application we find that the number of edges with
negative partial correlations is small and the numerical values of the negative partial correlations are
negligible. If the node has some edges with positive weights and some edges with negative weights, the
effects will cancel each other, resulting in small overall impact of the node.
coordinates \( \{b_1, \ldots, b_n\} \) and then recursively applying \( P \) we obtain

\[
P^{ke_0} = P^k \sum_{i=1}^n b_i u_i = \sum_{i=1}^n b_i P^k u_i = \sum_{i=1}^n b_i \lambda_i^k u_i = \lambda_1^k \sum_{i=1}^n b_i \left( \frac{\lambda_i}{\lambda_1} \right)^k u_i. \tag{8}
\]

Note that if \( \lambda_i/\lambda_1 < 1 \) for \( i > 1 \), all the terms in the last sum, with exception to the first, (equal to \( b_1 u_1 \)) tend to zero. In this case, therefore, the eigenvector of the largest eigenvalue, \( u_1 \), gives the asymptotic direction for \( P^{ke_0} \) when \( k \to \infty \). If we interpret \( e_0 \) as a vector of shocks as in Section 2.1 affecting the values of nodes in the graph, then the eigenvector centrality \( u_1 \) describes the asymptotic impact (\( k \)-th order when \( k \to \infty \)) of an initial shock on the nodes. The node with the largest component in the vector \( u_1 \) will receive the highest asymptotic impact of the shock.

Let us establish a connection between the eigenvector centrality of the nodes in the partial correlation network and the principle component analysis. To do this, we, first, generalize the notion of the eigenvector centrality. As we discussed, Eq. (8) implies that when \( \lambda_1 > \lambda_2 \geq \cdots \geq \lambda_n \), the eigenvector corresponding to \( \lambda_1 \) corresponds to the asymptotic distribution of the initial shocks over the nodes. The same formula makes clear, however, that the asymptotic behavior depends on the relative value of \( \lambda_1 \) with respect to the second largest eigenvalue \( \lambda_2 \) and, in general, to the whole spectrum.\(^5\)

For a given integer \( p < n \), we introduce the \( p \)-eigenvector centrality space of dimension \( p \) as the space spanned by the \( p \) eigenvectors of the adjacency matrix corresponding to the \( p \) largest eigenvalues. Intuitively, it is the subspace of \( \mathbb{R}^n \) where the shocks to the system would belong asymptotically in the case when the first \( p \) eigenvalues of the partial correlation matrix are large relative to the remaining eigenvalues. The approach used here resembles the principle component analysis (PCA) and the following proposition shows that it is, indeed, equivalent to the PCA applied to the matrix \( T \) defined in (7).

**Proposition 1.** Let \( \lambda \) be an eigenvalue of the adjacency matrix \( P \) of the graph of partial correlations with corresponding eigenvector \( u \). Then \( 1/(1 - \lambda) \) is the eigenvalue of matrix \( T \), the variance-covariance matrix of \( x \), defined in (7), with the same corresponding eigenvector \( u \).

\(^5\)For example, if \( \lambda_1 = \lambda_2 > \lambda_3 \) the asymptotic shock is in the subspace spanned by \( u_1 \) and \( u_2 \). It would be a mistake to characterize centrality by focusing only on the first eigenvector.
Proof. The statement follows from the following chain of equivalence relations:

\[ Pu = \lambda u \iff u - D_{K}^{-1/2}KD_{K}^{-1/2}u = \lambda u \iff \]
\[ \iff D_{K}^{-1/2}KD_{K}^{-1/2}u = (1 - \lambda)u \iff D_{\Sigma}^{-1/2}\Omega^{-1}D_{\Sigma}^{1/2}u = (1 - \lambda)u \iff \]
\[ \iff T^{-1}u = (1 - \lambda)u \iff \frac{1}{1 - \lambda}u = Tu. \]

The last Proposition asserts that matrix $P$ has the same eigenvectors as the rescaled variance-covariance matrix $T$, and even if their eigenvalues differ, their ordering is not. Therefore, the space where the first $p$ principle components of $T$ belong to is exactly the same as the $p$-eigenvector centrality space of $P$. In the special case of $p = 1$, we have that the usual eigenvector centrality of $P$ coincides with the first principle component of $T$.

Several generalizations of the eigenvector centrality have been proposed (Newman, 2010). Bonacich centrality measure reflects an idea that central nodes not only have a high degree centrality but also have neighbors with high degree centralities whose neighbors has high degree centrality as well and so on. If factor $\alpha$ measures the dampening in importance of degree centrality of the neighbors, then Bonacich centrality is defined as

\[ c^{B}(\alpha) = P \cdot 1 + \alpha P^{2} \cdot 1 + \alpha^{2}P^{3} \cdot 1 + \cdots = (I - \alpha P)^{-1}P \cdot 1, \tag{9} \]

where as before $1$ is an $n \times 1$ vector of ones and $I$ is the identity matrix of size $n$. When $\alpha = 0$ the Bonacich centrality is simply the degree centrality, i.e, the first-order effect of the unit shock $e = 1$. When $\alpha = 1$ the Bonacich centrality is equal to the cumulative of the first-, second-, and all higher order effects of the unit shock $e = 1$. Indeed, from Eq. (6) which gave the total effect, we have

\[ c^{B}(1) = (I - P)^{-1} \cdot 1 - 1. \]

Intermediate values of $\alpha$ are not particularly important in our application.\(^{7}\)

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\(^{6}\)The definition given here is a special case of the measure proposed in Bonacich (1987). There, for the network with adjacency matrix $A$ the centrality is defined as $c^{B}(\alpha, \beta) = \beta(I - \alpha A)^{-1}A \cdot 1$. The constant $\beta$ scales the centralities of all the elements and here we assume $\beta = 1$.

\(^{7}\)The Bonacich centrality measure was introduced for social networks where the edges are directly observed and the cumulative effect of the interactions is of interest. Dampening is a reasonable assumption for this setup. In our case, the network of partial correlations is obtained from the variance-covariance matrix, which already gives us the cumulative effect. The dampening is already accounted for in matrix.
4 Estimation procedure for time series

In Section 2 we have defined the network of partial correlations for an \( n \)-variate random variable. However, we are interested in using the network of partial correlations to describe cross-section linear dependence of an \( n \)-variate time series process.

Consider a stochastic process \( \{ Y_t : \Omega \to \mathbb{R}^n \}_{t=1}^T \), defined on a complete probability space \( (\Omega, \mathcal{F}, \mathbb{P}) \). The information set at time \( t \) is defined as \( \mathcal{F}_t = \sigma(Y'_1, \ldots, Y'_t) \). Next, we specify a multivariate conditional dynamic model

\[
Y_t = \mu_t(\theta_1) + \sqrt{H_t(\theta)} \varepsilon_t, \tag{10}
\]

where

\[
\mu_t(\theta_1) = (\mu_{1,t}(\theta_1), \ldots, \mu_{n,t}(\theta_1))' = \mathbb{E}[Y_t|\mathcal{F}_{t-1}]
\]

is a specification of the conditional mean, parametrized by a finite dimensional vector of parameters \( \theta_1 \), and

\[
H_t(\theta) = \text{diag}(h_{1,t}(\theta), \ldots, h_{n,t}(\theta)),
\]

where

\[
h_{i,t}(\theta) = h_{i,t}(\theta_1, \theta_2) = \mathbb{E}[(Y_{i,t} - \mu_{i,t}(\theta_1))^2|\mathcal{F}_{t-1}], \quad i = 1, \ldots, n,
\]

is the conditional variance of \( Y_{i,t} \) given \( \mathcal{F}_{t-1} \), parametrized by a finite-dimensional vector of parameters \( \theta_2 \), where \( \theta_1 \) and \( \theta_2 \) do not have common elements.

Assuming that the standardized innovations \( \varepsilon_t = (\varepsilon_{1,t}, \ldots, \varepsilon_{n,t})' \) are independent of \( \mathcal{F}_{t-1} \), that is, serially independent and identically distributed (i.i.d.), but cross-sectionally dependent, with constant correlation matrix \( \mathbf{R} \), we arrive to the constant conditional correlations (CCC) model of Bollerslev (1990).

The correlation matrix can be easily estimated using sample estimator,

\[
\hat{\mathbf{R}} = \frac{1}{T} \sum_{t=1}^T \varepsilon_t \varepsilon_t'.
\]

This estimator is equivalent to the MLE estimator. If the sample size \( T \) is small and the number of considered variables \( n \) is large, this sample correlation estimator becomes unstable. In this case shrinkage or penalized maximum likelihood estimators are handy. Given

\[\text{P.}\]

It is also possible to allow for time-varying correlations. One of the popular specification allowing for this is the dynamical conditional correlations (DCC) model of Engle (2002). We have implemented this specification as well, but the daily changes in partial correlations were very small relatively to the overall average level. The estimates we obtained with the DCC model for any given date were very similar to the CCC model.
that our sample size is sufficiently large relatively to the number of entities, we focus on the
sample correlations estimator. As a robustness check we also implement a shrinkage-based
glasso estimator of Peng et al. (2009) and report these results in Appendix E.

After obtaining the estimate of the correlation matrix, \( \hat{R} \) from the CCC model we use
Eq. (2) to obtain an estimate of the matrix of partial correlations.

5 Empirical application

The network setup described above may be applied in many different contexts. In this
Section we use this setup to uncover the perceived network of the Australian banks and indus-
tries. We use the term “perceived” to emphasize that our analysis is based solemnly on
the returns of publicly traded banks and sectors and can reveal the network of connections
implied only by the market-driven return co-movements.

Our sample spans the period from 6/11/2000 to 22/08/2014 and was obtained from
Datastream with 3,600 daily observations in total. We have identified 8 publicly traded
banks in the most recent period: the “big four” (ANZ, CBA, NAB, Westpac), two re-
gional banks (Bank of Queensland and Bendigo and Adelaide bank) and two large financial
groups providing banking services among others (Suncorp and Macquarie group). Next
we look at the index returns of two other major financial sub-sectors other then banking:
Insurance and Real Estate. In addition we include sectors of the real economy, Basic mater-
ials, Industrials and others. The sectoral classification is based on Industry Classification
Benchmark and is provided by Datastream (see Appendix B for details). We also include
the Asian market given its close links with the Australian economy.

We estimated the most typical specification in the CCC model assuming Gaussian
innovations, an ARMA(1,1) model for conditional mean equations and GARCH (1,1) for
conditional variance equations.

Initially the CCC model was estimated using the full 2000 – 2014 sample of 3,600 days
and later we divided the sample into two subsamples of roughly equal lengths: “pre 2008”
observations. These may be thought as pre-crises and post-crises subsamples, respectively.

We report the full matrix of correlations, \( R \), and partial correlations, \( P \), in Appendix C.
There are multiple instances when two entities with relatively high correlation have low
Figure 1: Network of partial correlations of the Australian banks and other sectors. Full sample of 3,600 days was used for estimation.
Figure 2: Evolution of the network of the partial correlations of the Australian banks and other sectors.

partial correlation. For example, the correlation between NAB and Westpac is 0.65, whereas the partial correlation is only 0.1. This is an indication that NAB and Westpac have relatively strong indirect connections.

Figure 1 shows the reconstructed network of partial correlations. The thickness of the edges corresponds to the strength of the partial correlations. For improved visibility we do not show edges corresponding to partial correlations smaller than 0.075. The banks and other financials are collected in the lower part of the graph. We notice strong partial correlations between the big four banks and their links with other banks, financial and real sector. Interestingly, Macquarie group links the banking sector with the real economy and Asia. ANZ and Bank of Queensland have direct strong links with the Real Estate subsector. The regional banks, Bank of Queensland and the Bendigo and Adelaide bank are strongly linked directly, the former also has a link with NAB, while the latter has a strong link with the Macquarie group and Suncorp. Industrials and Consumer Svs sectors seems to be in the center of the Australian economy. Asian market influences the Australian economy primarily through the Basic materials and Oil & Gas and Consumer Svs.

---

9We also considered other major international markets European and North American market. However, the trading hours of the Australian market do not overlap with these markets. The impact of Australian market on these markets was transmitted via the Asian market. When we considered the impact of the North American and European markets on the Australian market again the transmission was mainly coming through the Asian market, the Basic material were effected directly.
Figure 2 shows the networks of partial correlations for pre 2008 and post 2008 subsamples. For convenience we kept the locations of the nodes fixed. We notice that some connections have changed substantially pre and post 2008. Note that the estimates for subsamples are subject to higher estimation noise due to smaller sample size compared to the full sample. Interestingly, the interbank connections and connections between the banks and other sectors have increased post 2008. At the same time we note a decrease in the central role of mining sector post 2008 and increased central role of the industrials.

In addition to graphical representations, we compute various network-based centrality measures which help to identify the most important nodes. Table 1 reports these measures for all economic entities. The first measure is $R^2$ as defined in (3), which is the proportion of variation in the returns explained by the returns of all other components in the network. The remaining measures are the degrees centrality, the eigenvector centrality and the Bonacich $\alpha$-centrality with $\alpha = 1$, discussed in Section 3. Next to the measure we report its ranking in the descending order. All measures are reported for the full sample and for Bonacich centrality we additionally report the values for pre and post 2008 samples (the other measures for these periods are reported in Appendix D). The overall ordering

<table>
<thead>
<tr>
<th>Measure</th>
<th>Sample</th>
<th>$R^2$</th>
<th>Degree</th>
<th>Eigenv.</th>
<th>Bonacich full</th>
<th>Bonacich pre 2008</th>
<th>Bonacich post 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANZ</td>
<td>full</td>
<td>0.660 1</td>
<td>1.074 4</td>
<td>1.000 1</td>
<td>23.481 1</td>
<td>13.212 2</td>
<td>32.294 3</td>
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<tr>
<td>Westpac</td>
<td>full</td>
<td>0.646 2</td>
<td>1.056 6</td>
<td>0.968 2</td>
<td>22.759 3</td>
<td>12.454 4</td>
<td>31.993 4</td>
</tr>
<tr>
<td>Industrials</td>
<td>full</td>
<td>0.621 4</td>
<td>1.363 1</td>
<td>0.967 3</td>
<td>23.478 2</td>
<td>12.111 7</td>
<td>40.957 1</td>
</tr>
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<td>full</td>
<td>0.618 5</td>
<td>1.063 5</td>
<td>0.932 4</td>
<td>21.993 4</td>
<td>11.319 10</td>
<td>31.947 5</td>
</tr>
<tr>
<td>CBA</td>
<td>full</td>
<td>0.609 6</td>
<td>0.995 8</td>
<td>0.906 5</td>
<td>21.333 5</td>
<td>12.362 6</td>
<td>29.240 6</td>
</tr>
<tr>
<td>Basic Materials</td>
<td>full</td>
<td>0.623 3</td>
<td>1.121 3</td>
<td>0.877 6</td>
<td>21.189 6</td>
<td>15.276 1</td>
<td>28.138 8</td>
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<tr>
<td>Consumer Svs</td>
<td>full</td>
<td>0.557 8</td>
<td>1.221 2</td>
<td>0.831 7</td>
<td>20.258 7</td>
<td>11.858 8</td>
<td>34.707 2</td>
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<tr>
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<td>full</td>
<td>0.578 7</td>
<td>0.971 9</td>
<td>0.795 8</td>
<td>19.204 8</td>
<td>11.484 9</td>
<td>28.156 7</td>
</tr>
<tr>
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<td>1.010 7</td>
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<td>17.975 9</td>
<td>12.493 3</td>
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<td>Macquarie</td>
<td>full</td>
<td>0.469 10</td>
<td>0.851 10</td>
<td>0.713 10</td>
<td>17.031 10</td>
<td>10.238 11</td>
<td>23.117 11</td>
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<tr>
<td>Asia Market</td>
<td>full</td>
<td>0.451 11</td>
<td>0.793 13</td>
<td>0.665 11</td>
<td>15.993 11</td>
<td>9.161 13</td>
<td>24.890 9</td>
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<td>0.813 11</td>
<td>0.615 12</td>
<td>14.818 12</td>
<td>9.679 12</td>
<td>19.196 15</td>
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<tr>
<td>Suncorp</td>
<td>full</td>
<td>0.377 13</td>
<td>0.740 17</td>
<td>0.591 13</td>
<td>14.135 13</td>
<td>8.888 14</td>
<td>18.485 16</td>
</tr>
<tr>
<td>Bank of Qld.</td>
<td>full</td>
<td>0.365 14</td>
<td>0.741 16</td>
<td>0.570 14</td>
<td>13.676 14</td>
<td>7.164 15</td>
<td>19.419 14</td>
</tr>
<tr>
<td>Bend&amp;Ad.Bank</td>
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<td>0.365 15</td>
<td>0.799 12</td>
<td>0.564 15</td>
<td>13.597 15</td>
<td>6.801 16</td>
<td>20.748 13</td>
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<tr>
<td>Utilities</td>
<td>full</td>
<td>0.350 17</td>
<td>0.766 15</td>
<td>0.543 16</td>
<td>13.210 16</td>
<td>6.680 17</td>
<td>21.128 12</td>
</tr>
<tr>
<td>Health Care</td>
<td>full</td>
<td>0.351 16</td>
<td>0.787 14</td>
<td>0.526 17</td>
<td>12.874 17</td>
<td>12.373 5</td>
<td>14.632 18</td>
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<tr>
<td>Consumer Gds</td>
<td>full</td>
<td>0.217 18</td>
<td>0.504 18</td>
<td>0.363 18</td>
<td>8.888 18</td>
<td>6.168 18</td>
<td>17.513 17</td>
</tr>
<tr>
<td>Technology</td>
<td>full</td>
<td>0.159 20</td>
<td>0.432 20</td>
<td>0.306 19</td>
<td>7.472 19</td>
<td>4.711 20</td>
<td>11.205 19</td>
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<tr>
<td>Telecom</td>
<td>full</td>
<td>0.167 19</td>
<td>0.464 19</td>
<td>0.304 20</td>
<td>7.453 20</td>
<td>5.472 19</td>
<td>9.764 20</td>
</tr>
</tbody>
</table>
is based on eigenvector centrality for the full sample. We notice that the ranking for the full sample is similar for all the considered measures. The eigenvector centrality and the Bonacich centrality measures have the closest ranking similarity with only one different entry. Using all four measures ANZ seems to have the most central position. The big four banks together with Industrials have the highest centrality using the full sample. This indicates the importance of the banking sector in shock transmission. When we compare the pre and post 2008 periods we notice a significant increase in the network effects. The average $R^2$ of all entities increased from 0.34 pre 2008 to 0.54 post 2008. Similarly the average Bonacich centrality increased from 10 to 24. Interestingly, Basic materials showed the highest centrality pre 2008, and Industrials became the most central post 2008. The big four banks remained relatively highly central in both periods, however the levels of Bonacich centrality increased dramatically post 2008 indicating higher network effects of the banks.

Finally, let us briefly discuss the results based on glasso method of Peng et al. (2009) reported in Appendix E. Due to shrinkage we notice substantially lower values of correlations and somewhat reduced partial-correlations in comparison to the baseline case of sample correlations estimator. However, the centrality-based ranking are very similar to the baseline case. We conclude that while we may overestimate the effect of the partial correlations using the sample correlations estimates, the centrality ranking will not be strongly effected.

5.1 Policy examples

Suppose a policy-maker wishes to lower the total effect of a shock to the economic system in the most effective way. We can measure the total effect of a unit exogenous shock affecting all entities to any specific entity in the system with the Bonacich centrality measure. Using the centrality measures in Table 1 it is easy to identify that ANZ is the most central in the system (based on the full sample). Let assume that a policy-maker implements a set of measures reducing the partial correlations of all entities connected with ANZ by 10 percent. We compute that in this case the Bonacich centrality measure of ANZ will be reduced by 33 percent from about 23 to 15.3. Moreover the average Bonacich centrality of all big four banks will be reduced by 31 percent from 22.4 to 15.4, while the average Bonacich centrality of the system will be reduced by 24 percent from 16.5 to 12.5.
Now, let us check what will happen if a policy maker were to focus on important, but a less central entity, say Macquarie group and reduced its partial correlations by 10 percent. The Bonacich centrality of the Macquarie group would reduce by 23 percent, the average Bonacich centrality of the big four banks would be reduced by 15 percent and the average Bonacich centrality of the whole system would be reduced by 15 percent, which is smaller compared to the optimal policy targeting the most central entity. Focusing on the least central entity such as Telecom sector and reducing all its connections by 10 percent would reduce the average Bonacich centrality of the system only by 3.5 percent.

It is important to mention that the policy examples considered above are rather stylized and are given just as an illustration of possible use for the discussed centrality measures. We have only considered the benefits of the policy. However, costs of reducing the connections of a highly central entity may be substantially higher compared to the cost of reducing the connections of less central entity. The centrality measures along with the networks of partial correlations may be used as complimentary indicators guiding policy makers.

6 Concluding remarks

In this paper we linked various methods for reconstruction of the partial correlations networks, established the connections between the theoretical network measures and principal component analysis and applied the methodology to reconstruct the implied networks of partial correlations between the relative change in value of the Australian banks, other domestic sectors and international markets. We investigated the evolution of the networks over time and computed network-based measures for the considered entities.

We found strong direct links between the big four Australian banks and their connection to the real economy via financial services. We also observed that the Australian economy links to the international markets via the Asian market.

References


Allen, F., Babus, A., 2009. Networks in finance, in: Kleindorfer, P.R., Wind, Y.J.R., Gun-


A Properties of Partial Correlations

This Appendix collects the results on partial correlations which are used in this paper; most of them are mentioned in Section 2. Many results about partial correlations can be found in Chapter 13 of Bühlmann and Van De Geer (2011), Chapter 17 of Hastie et al. (2009) and Chapter 5 of Whittaker (2009). We selected here the results we need in this paper and adopted them to our notation.

First, we reproduce the definition. Let \( X \) be the multivariate random variable with components \( X_1, \ldots, X_n \). Let \( X_{i\cdot} \) denote the best linear approximation of variable \( X_i \) based on all the variables except for \( X_i \) and \( X_j \) for any pair \( i, j \).

**Definition.** The partial correlation coefficient between \( X_i \) and \( X_j \), denoted by \( \rho_{ij} \), is defined as the ordinary correlation coefficient between \( X_i - X_{i\cdot} \) and \( X_j - X_{j\cdot} \).

Without loss of generality we can assume that all the components of \( X \) have zero mean.

For the sake of notation simplicity, the proofs will be focused on random vectors \( X_1 \) and \( X_2 \). We denote \( \epsilon_{12} = X_1 - X_{12\cdot} \) and \( \epsilon_{21} = X_2 - X_{21\cdot} \), the components of \( X_1 \) and \( X_2 \) orthogonal to the space spanned by the remaining random variables \( X_3, \ldots, X_n \). By definition the partial correlation coefficient between \( X_1 \) and \( X_2 \) is

\[
\rho_{12} = \frac{E(\epsilon_{12}\epsilon_{21})}{\sqrt{E\epsilon_{12}^2 E\epsilon_{21}^2}}.
\]

A.1 Connection with Linear Regression

To get representation \( (E) \) we project \( X_1 \) on all the remaining components of \( X \) and also project \( X_2 \) on all the remaining components of \( X \) (recall that in this appendix the variables are already transformed to have zero mean):

\[
\begin{align*}
X_1 &= \beta_{12}X_2 + \beta_{13}X_3 + \cdots + \beta_{1n}X_n + \epsilon_1 \\
X_2 &= \beta_{21}X_1 + \beta_{23}X_3 + \cdots + \beta_{2n}X_n + \epsilon_2.
\end{align*}
\] (11)

We will assume that the components of \( X \) are linearly independent and so the projection errors \( \epsilon_1 \) and \( \epsilon_2 \) are not constant. The next result establishes the link between \( E\epsilon_{12}^2 \), the variance of the projection error when \( X_1 \) is projected on the space spanned by \( X_3, \ldots, X_n \), and \( E\epsilon_1^2 \), the variance of the projection error when \( X_1 \) is projected on the space spanned by \( X_2, X_3, \ldots, X_n \). The latter is called the conditional variance of \( X_1 \) in the main text.

**Lemma.** Assume the orthogonality conditions \( E(\epsilon_1 X_i) = 0 \) for all \( i \neq 1 \) and \( E(\epsilon_2 X_i) = 0 \) for all \( i \neq 2 \). Then

\[
E\epsilon_{12}^2 = \frac{E\epsilon_1^2}{1 - \beta_{12}\beta_{21}}.
\]

**Proof.** Using several times the orthogonality of \( \epsilon_{12} \) to the space spanned by \( X_3, \ldots, X_n \) and substituting \( X_1 \) and \( X_2 \) from (11), we obtain that

\[
E\epsilon_{12}^2 = E(X_1\epsilon_{12}) = E((\beta_{12}X_2 + \epsilon_1)\epsilon_{12}) = \beta_{12}E(X_2\epsilon_{12}) + E\epsilon_1^2 = \beta_{12}E((\beta_{21}X_1 + \epsilon_2)\epsilon_{12}) + E\epsilon_1^2 = \beta_{12}\beta_{21}E\epsilon_{12}^2 + E\epsilon_1^2.
\]

At the last step we used \( E(\epsilon_2 \epsilon_{12}) = 0 \) which holds due to the orthogonality conditions for \( \epsilon_2 \). Note that \( \epsilon_{12} = X_1 - X_{12\cdot} \) is a linear combination of \( X_1, X_3, \ldots, X_n \).
The required result follows now from the equality derived above. Note that $\beta_{12}\beta_{21} \neq 1$, as otherwise $E\varepsilon_1^2 = 0$. But this would contradict the assumption of linear independence of $X_1, \ldots, X_n$. \hfill \square

**Proposition 2.** Assume the orthogonality conditions $E(\varepsilon_1 X_i) = 0$ for all $i \neq 1$ and $E(\varepsilon_2 X_i) = 0$ for all $i \neq 2$ in (11). Then

$$\beta_{12} = \rho_{12}; \sqrt{\frac{E\varepsilon_1^2}{E\varepsilon_1^2}} = \rho_{12}; \sqrt{\frac{\operatorname{Var}(\varepsilon_1)}{\operatorname{Var}(\varepsilon_2)}}$$

**Proof.** Let’s compute $E(\varepsilon_1 \varepsilon_{21})$. On the one hand, we can express $\varepsilon_1$ from (11) and due to orthogonality of $\varepsilon_{21}$ to $X_3, \ldots, X_n$ we get

$$E(\varepsilon_1 \varepsilon_{21}) = E(X_1 \varepsilon_{21}) - \beta_{12} E(X_2 \varepsilon_{21}).$$

On the other hand, $\varepsilon_{21} = X_2 - X_2^*$, is a linear combination of $X_2, \ldots, X_n$ and $\varepsilon_1$ is orthogonal to all these vectors. Hence, $E(\varepsilon_1 \varepsilon_{21}) = 0$ and therefore

$$\beta_{12} = \frac{E(X_1 \varepsilon_{21})}{E(X_2 \varepsilon_{21})} = \frac{E((X_1^* + \varepsilon_{12}) \varepsilon_{21})}{E((X_2^* + \varepsilon_{21}) \varepsilon_{21})} = \frac{E(\varepsilon_{12} \varepsilon_{21})}{E\varepsilon_{21}^2} = \rho_{12}; \sqrt{\frac{E\varepsilon_{12}^2}{E\varepsilon_{21}^2}},$$

where at the last step we used the definition of the partial correlation. The previous lemma implies that the ratio of variances of $\varepsilon_{12}$ and $\varepsilon_{21}$ is the same as the ratio of variances of $\varepsilon_1$ and $\varepsilon_2$. It completes the proof. \hfill \square

From the last Proposition by symmetry we derive that

$$\beta_{12}\beta_{21} = \rho_{12}^2 \quad \Leftrightarrow \quad \rho_{12} = \frac{\operatorname{sign}(\beta_{12})}{\sqrt{\beta_{12}\beta_{21}}},$$

Since correlation coefficient is always between $-1$ and $1$, it follows (see the previous Lemma) that $\beta_{12}\beta_{21} < 1$ and that $E\varepsilon_1^2 < E\varepsilon_{21}^2$.

These results hold for any $i$ and $j$ and allow us to establish a link between the matrix of partial correlations, $\mathbf{P}$, and the matrix of linear coefficients in system (E), $\mathbf{B}$. (Both matrices have zeros on the diagonal.) In the main text we defined the diagonal matrix $\mathbf{D}_\Sigma = \operatorname{diag}\{\operatorname{Var}(\varepsilon_1), \ldots, \operatorname{Var}(\varepsilon_n)\}$. The result of Proposition 2 implies that $\mathbf{B} = \mathbf{D}_\Sigma^{1/2} \mathbf{PD}_\Sigma^{-1/2}$. With this result we can directly obtain system (5) which played a crucial role in our separation of the first-order effect of the shock from the total effect of the shock.

**A.2 Connection with Concentration Matrix**

To obtain a useful characterization of partial correlation, we investigate the elements of the concentration matrix $\mathbf{K} = \mathbf{\Omega}^{-1}$. Without loss of generality, we will focus only on the first row and the first column of this matrix.

Let $\mathbf{X}$ denote the matrix corresponding to the multivariate random variable $X$. The columns of $\mathbf{X}$ are the random vectors $X_1, \ldots, X_n$. Let $\mathbf{X}_{-1}$ denote the matrix whose columns are the random vectors $X_2, \ldots, X_n$. Consider the first equality in (11), and write it as $X_1 = \mathbf{X}_{-1} \beta + \varepsilon_1$, where $\beta = (\beta_{12} \cdots \beta_{1n})^T$. Using the orthogonality condition, we obtain from the normal equations that

$$\beta = (E(\mathbf{X}_{-1}^T \mathbf{X}_{-1}))^{-1} E(\mathbf{X}_{-1}^T \varepsilon_1).$$

The next result exploits the block structure of the variance-covariance matrix of $X$. 

\[ \text{22} \]
Lemma. Variance-covariance matrix of $X$ can be represented as follows

$$\Omega = E(X^TX) = \begin{pmatrix} 1 & \beta^T \\ 0 & I \end{pmatrix} \begin{pmatrix} \text{Var}(\varepsilon_1) & 0 \\ 0 & E(X^T_1X_{-1}) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \beta & I \end{pmatrix}$$

Proof. This can be checked by direct computation. For example, variance of $X_1$ is

$$E(X_1^TX_1) = E(\beta^TX^T_1\varepsilon_1^T(X_{-1}\beta + \varepsilon_1)) = \beta^TE(X^T_1X_{-1})\beta + \text{Var}(\varepsilon_1)$$

which is exactly the upper left element from the right-hand side of the equality. Also, the row vector of covariances of $X_1$ with the remaining vectors is

$$E(X_1^TX_{-1}) = E(\beta^TX^T_1\varepsilon_1^T(X_{-1}) = E(\beta^TX^T_1X_{-1})$$

which coincides with the upper right element of the right-hand side.

Using the decomposition of $\Omega$ derived in the previous Lemma we can express the concentration matrix, $K = \Omega^{-1}$ in the block form as well:

$$\Omega^{-1} = \begin{pmatrix} 1 & 0 \\ \beta & I \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{\text{Var}(\varepsilon_1)} & 0 \\ 0 & (E(X^T_1X_{-1}))^{-1} \end{pmatrix} \begin{pmatrix} 1 & \beta^T \\ 0 & I \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ -\beta & I \end{pmatrix} \begin{pmatrix} \frac{1}{\text{Var}(\varepsilon_1)} & 0 \\ 0 & (E(X^T_1X_{-1}))^{-1} \end{pmatrix} \begin{pmatrix} 1 & -\beta^T \\ 0 & I \end{pmatrix} = \begin{pmatrix} \frac{1}{\text{Var}(\varepsilon_1)} - \beta^T \frac{1}{\text{Var}(\varepsilon_1)} \beta & \frac{1}{\text{Var}(\varepsilon_1)} \beta^T + (E(X^T_1X_{-1}))^{-1} \end{pmatrix}$$

Studying the elements in the first row of this matrix we obtain the following results for the concentration matrix. The upper left element, $k_{11} = 1/\text{Var}(\varepsilon_1)$. The second element in the first row $k_{12} = -\beta_12k_{11}$ which using Proposition 2 can be rewritten as

$$k_{12} = -\beta_12k_{11} = -\rho_{12}\sqrt{\frac{\text{Var}(\varepsilon_1)}{\text{Var}(\varepsilon_2)}}k_{11} = -\rho_{12}\sqrt{k_{11}k_{22}}.$$ Expressing the partial correlation from the equation we obtain an instance of (2). In general, the following holds.

Proposition 3. The on-diagonal elements of the concentration matrix $K$ are given by

$$k_{ii} = \frac{1}{\text{Var}(\varepsilon_i)}.$$ (13)

The off-diagonal elements of the concentration matrix $K$ are proportional to the negative of the corresponding partial correlations. Namely, Eq. (2) holds

$$\rho_{ij} = -\frac{k_{ij}}{\sqrt{k_{ii}k_{jj}}}.$$ The last result in the matrix form is Eq. (4).

A.3 Connection with the Inverse of Correlation Matrix

We use the matrix notation. Let $D_\Omega$ be the diagonal matrix composed of the diagonal elements of $\Omega$. Then, by definition, the correlation matrix of $X$, denoted as $R$ can be
written as $\mathbf{R} = \mathbf{D}_\Omega^{-1/2} \mathbf{\Omega} \mathbf{D}_\Omega^{-1/2}$. Therefore,

$$\mathbf{R}^{-1} = \mathbf{D}_\Omega^{1/2} \mathbf{K} \mathbf{D}_\Omega^{1/2}.$$ 

Thus, $(i,j)$ element of matrix $\mathbf{R}^{-1}$ is $k_{ij} \sqrt{\text{Var}(X_i) \text{Var}(X_j)}$. With such proportionality to the elements of $\mathbf{K}$, the partial correlations can also be computed if in Eq. (2) the elements of $\mathbf{K}$ are substituted by the elements of $\mathbf{R}^{-1}$. Using (13) we find that the diagonal elements of $\mathbf{R}^{-1}$ are $\text{Var}(X_i)/\text{Var}(\varepsilon_i)$. 

24
B Industry Classification Benchmark

We adopted the following sectoral classification from Datastream which is based on the Industry Classification Benchmark:

- Oil & Gas
  - Oil and Gas Producers
  - Oil Equipment, Services and Distribution
- Basic Materials
  - Chemicals
  - Basic Resources including Mining and Industries Metals
- Industrials
  - Construction and Materials
  - Industrial Goods and Services including transportation and business support
- Consumer Goods
  - Food and Beverages
  - Personal and Household Goods including Home Construction
- Health Care
  - Health Care Equipment and Services
  - Pharmaceuticals and Biotechnology
- Consumer Services
  - Retail
  - Media
  - Travel and Leisure
- Telecommunications
- Utilities
  - Electricity
  - Gas, Water and Multi-Utilities
- Financials
  - Banks
  - Insurance
  - Real estate including real estate investment and services and trusts
  - Financial services including financial groups
- Technology
  - Software and Computer Services
C Correlations and partial correlations

Below we report matrices of correlations and partial correlations for the full sample.

Table 2: Matrix of correlations

<table>
<thead>
<tr>
<th></th>
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D Centrality measures of pre and post 2008 samples

Below we compare centrality measures of pre and post 2008 samples.

Table 4: Centrality Measures for the Network of Partial Correlations based on pre 2008 sample

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Table 5: Centrality Measures for the Network of Partial Correlations based on post 2008 sample

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E Glasso-based estimates and centrality measures

As a robustness check we estimated the matrices of correlations and partial correlations using the glasso method of Peng et al. (2009). The glasso method exploits the relationship between partial correlations and the system of regression equations in Eq. shrinking the parameters towards zero. The methods relies heavily on the choice of regularization parameter. We rotation information criterion the choice of the latter. The estimation was implemented in R using package ‘huge’.

Below we report matrices of correlations and partial correlations for the full sample.

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