This research was supported by the Centre for International Finance and Regulation, which is funded by the Commonwealth and NSW Governments and supported by other Consortium members.
Systemic Financial Risk Inference in a Global Setting

Jeffrey Sheen, Stefan Trück, Chi Truong, Ben Z Wang

Faculty of Business and Economics, Macquarie University, NSW, Australia, 2109

Abstract

We propose a new top-down approach to measure systemic risk in the financial system. Our framework uses a combination of macroeconomic, financial and rating factors in representative regions of the world. We formulate a mixed-frequency state-space model to estimate macroeconomic factors. To derive financial risk factors, we use Moody’s/KMV expected default frequencies after accounting for ratings of major financial institutions in the considered regions. The estimated factors are combined to derive probabilities for systemically relevant defaults in the financial industry. Regional macroeconomic factors are significant predictors of the existence and number of systemically important defaults, while regional financial risk and ratings factors are relevant for the existence only. For major events, global credit risk also matters.

Keywords: Systemic financial risk, Factor models, Mixed frequency models, Kalman filter, State-space model, Hurdle model

JEL Codes: C33, E44, G01, G17.

We would like to thank the Australian Research Council and the Centre for International Finance and Regulation for providing funding support on DP 120102220 and CIFR-E034.

Email addresses: jeffrey.sheen@mq.edu.au (Jeffrey Sheen), stefan.trueck@mq.edu.au (Stefan Trück), chi.truong@mq.edu.au (Chi Truong), ben.wang@mq.edu.au (Ben Z Wang)

July 10, 2014
1. Introduction

We introduce a new top-down approach to measure systemic risk in the financial system, combining information on macroeconomic, financial and ratings factors in four representative regions of the world. We find that regional macroeconomic conditions can predict the existence of a systemically important financial events, as well as the number within a period. A measure of regional financial risk as well as a ratings factor are also significant predictors for the existence of systemic events, and a measure of a global credit risk premium is associated with major events. First we formulate a state space dynamic factor model (DFM) of a variety of observed macroeconomic and financial variables from the United States, the European Union, Australia and China since 1990. We estimate unobserved macroeconomic factors for the ‘world’ and for each representative region, using observed stock and flow variables arriving at mixed frequencies. At the next stage, these macroeconomic factors are used as drivers along with risk ratings to help explain financial risks in each region, derived from Moody’s/KMV expected default frequencies for major financial institutions in each region. Finally, the macroeconomic, financial and ratings factors are then combined to derive probabilities for systemically relevant defaults in the financial industry. We then infer the evolution of systemic risks in each region from these probability estimates using a hurdle model, yielding local predictions from global information.

The need for a better understanding of the drivers of financial risk in a systemic context became a major issue as a result of the recent global financial crisis in 2007 and 2008. In response to the severe cost of the crisis, the U.S. Congress enacted the Dodd Frank Wall Street Reform and Consumer Protection Act, usually referred to as the Dodd Frank Act in July 2010. As a major part of this reform, the Financial Stability Oversight Council (FSOC) was established to identify risks to financial stability arising from events or activities of large financial firms or elsewhere and to respond to emerging threats to the stability of the financial system. The Dodd Frank Act is evidence of the emphasis regulators put on identifying and reacting to potential threats of the financial system as a whole.

Research on financial crises and systemic risk has grown rapidly during the last 5 years, for example, see Ishikawa et al. (2012) for a summary of recent empirical literature and Bisias et al. (2012) for a survey on quantitative approaches to the measurement of systemic risks. Generally, the starting point for monitoring and responding to potential systemic risks is the timely and accurate measurement of threats to the financial system. The economic and financial literature provides a variety of approaches to monitoring these risks, including models based on measures of illiquidity, default risk and probability distributions, on measures created from network analysis and graph-theoretic techniques, and on macroeconomic measures (Bisias et al. 2012). The suggested methods fall roughly into two categories: bottom-up and top-down approaches.

Bottom-up approaches in general measure the contribution of individual institutions to the systemic risk of the entire financial market or system in a region. They often also measure feedback effects between shocks to the financial system and the risk of individual financial institutions. Important representative work in this area includes Acharya et al. (2012b), Adrian and Brunnermeier (2011), Allen et al. (2012), and Brownlees and Engle (2012), just to mention a few.

Adrian and Brunnermeier (2011) introduce the Delta Conditional Value-at-Risk (CoVaR) measure, relating systemic risk to the Value-at-Risk (VaR) of the market conditional on individual
institutions being under distress. Similarly, [Hautsch et al. (2011)](#) measure systemic risk as the
time-varying contribution of a firm’s VaR on the market VaR, while [White et al. (2012)](#) concen-
trate on spillover effects between the VaR of a financial institution and the market. [Allen et al. (2012)](#) derive a measure of aggregate systemic risk (CATFIN) using the 1% VaR measures of a
cross-section of financial firms. They suggest that the derived measure forecasts economic
downturns almost one year in advance in conducted out-of-sample tests.

Acharya et al. (2012b) measure systemic risk of a financial institution as its contribution to the
total capital shortfall of the financial system that could be expected in a future crisis and de-
rive the so-called Marginal Expected Shortfall (MES) and Systemic Expected Shortfall (SES).
Following this line of thought, Acharya et al. (2012a); Brownlees and Engle (2012) propose a
systemic risk measure (SRISK) that captures the expected capital shortage of a firm given its
degree of leverage and marginal expected shortfall. Usually, these studies measure the contribu-
tion of major US financial institutions to systemic risk. Note that [Benoit et al. (2013)](#) provide
a theoretical and empirical comparison of several of these approaches and find that different
systemic risk measures identify very different financial institutions as being systemically im-
portant. Also, their results suggest that rankings based on systemic risk estimates often mirror
rankings that could be obtained by sorting the firms based on market risk or liabilities.

Top-down approaches usually rather identify systemic risk by inferring factors relating to high
level features of the financial system, potentially also including macroeconomic variables.
Lowe and Borio (2002) and Borio and Lowe (2004) explain how systemic financial distress
often arises because financial imbalances develop in otherwise benign circumstances. For 34
countries in 1960-1999, they find that sustained credit and asset price growth increased financial
instability risk. [Billio et al. (2012)](#) use principal components to analyze the interconnectedness
among hedge funds, banks, brokers, and insurance companies. They find high interrelatedness
between these, which have become less liquid in recent years, increasing the level of systemic
risk particularly for the finance and insurance industries. [Allen et al. (2010)](#) examine the impact
of networks and the architecture of the financial system on systemic risk. [Schwaab et al. (2014)](#)
apply so-called coincident risk measures and early warning indicators for financial distress of
the whole system, derived from macro and credit risk data.

In this paper we develop a top-down approach to create early warning indicators for systemic
risk. Our research design has three key elements: a global macroeconomic DFM, a global fi-
nancial risk DFM, and a global systemic risk hurdle regression model.

In the first two elements, we propose DFMs for the measurement of global macroeconomic and
financial conditions based on state space methods. In particular we estimate mixed frequency
models to create real-time indicators of macro-financial and credit risk conditions. The applied
framework follows work by Mariano and Murasawa (2003); Aruoba et al. (2009) who apply
mixed-frequency models to extract factors that summarize various sources of information arriv-
ing at different frequency. Factor models and the dimension reduction of a set of explanatory
variables have always played a substantial role in the analysis of financial markets. Applica-

\[1\]Our approach relates to the work on now-casting and real-time business and financial condition indicators.
Recently, various authors have explained the importance of measuring financial and economic activity at high
frequency, for example see [Altissimo et al. (2001); Evans (2005); Giannone et al. (2008); Angelini et al. (2011)].
In this paper, we do not restrict our attention to observable variables at the same frequency, for example, as
tions include, for example, asset pricing, the analysis of risk and returns, portfolio management and modelling term structure dynamics. They allow a reduction in the dimensionality of the set of potential explanatory variables, leading to parsimonious and efficient risk management tools. DFMs have become popular in macroeconomics and finance because they provide a powerful tool for understanding the co-movement between many time series (see for example Geweke (1977), Bernanke et al. (2005), Kose et al. (2012) and Leu and Sheen (2011)). Using observed macroeconomic and other financial variables, e.g. forward-looking measures such as implied volatilities, in a DFM, we estimate unobserved factors for global business cycle conditions (common to all observed variables) as well as region-specific business cycle conditions (which are common to all observed variables in a particular region). Following a similar approach for the second element, we use observed measures of future credit risk in the form of expected default frequencies (EDFs) for a set of the top 11 financial companies in each region to estimate unobserved factors for region or country-specific financial hazards (common to all credit risk variables in a specific region) after accounting for ratings by Moody’s and our macroeconomic factors. These financial factor estimates deliver model-based predictors of financial distress, beyond what ratings agencies produce. The interesting question is what these model-based predictors can explain just before and during systemic crises, and whether they can be used to monitor and forecast systemic risks globally or in a specific region.

Accordingly, in our third element, we delve further into early warning systems of financial stress and default, integrating our DFM factor estimates with actual default events. We contribute to the literature on systemic risk quantification, in particular to the area of top-down approaches for the measurement of systemic risks in the financial system. As complements to information on actual defaults and EDFs, we also include other measures of credit risk, such as factors for credit ratings provided by Moody’s KMV and TED spreads. Since actual defaults are relatively rare, the monthly time-series of default counts is heavily inflated by zeros. Having zero or not zero defaults in a month may well be determined by different covariates than the actual number of non-zero defaults. Recognizing this, a major contribution of this paper is that we employ a hurdle model to explain the zero counts (with a Binomial process) and the non-zero counts (with a Poisson process) using different explanatory variables. This distinction is important because we find only recent macroeconomic history helps in explaining the severity of a crisis (the number of defaults), while the financial, macroeconomic and credit rating factors are all important in explaining the existence of a crisis (the zeros). Also this distinction enables us to compare the determinants of low hurdle with high hurdle crises.

We use data for variables that are available at different frequencies, but estimate all unobservable factors at the highest frequency of the observable data. To create an effective framework for delivering metrics of economic activity at monthly frequency in a dynamic factor model with data on relevant variables arriving at different frequencies, the set of information needs to be integrated using some type of efficient multivariate filter. We apply a mixed-frequency approach with the Kalman filter as in Mariano and Murasawa (2003), Aruoba et al. (2009), Sheen et al. (2011) to integrate macroeconomic and financial variables in a coherent framework. Since this approach leads to missing data for variables at low frequencies, a state-space representation is appropriate. For the estimation of these models, the filter delivers high-frequency unobserved indicators, which utilises efficiently information from the lower-frequency variables despite many missing data points.

2In this line of research, usually a combination of high-level features of the macro economy and the financial system are used to create forward-looking measures of systemic risk. Giesecke and Kim (2011) use a hazard rate approach with contagion and additional macro-financial factors as exogenous regressors to measure systemic risk. Schwaab et al. (2014) use Moody’s credit risk data alongside macro and financial data for the US and EU (and a mixture of countries for the rest of the world) to construct financial failure indicators.
Another contribution of our paper is that we provide a framework for measuring and analyzing drivers of systemic risks for the financial sector across different geographical regions using our hurdle model. So far only a limited number of studies have focussed on an international or global perspective. Exceptions include the work by [Pesaran et al. (2006); Schwaab et al. (2014)]. By taking an international perspective and identifying global macroeconomic and financial factors as well as links between the considered regions, the developed framework also allows us to retrieve information about systemic risk for regions where only limited (or even no) information exists in default or ratings data for the financial sector. For example, for a country like Australia, where the number of observed defaults for financial institutions is very low and only a limited number of companies are rated by the major ratings agencies (such as Moody’s/KMV), it will be virtually impossible to derive appropriate systemic risk indicators from local data. By taking a global perspective, we can at least partially overcome this problem and assess systemic risk for such a country with limited information by taking advantage of international linkages and systematic credit risk conditions across different geographical regions.

The remainder of the paper is organized as follows. Section 2 introduces the applied framework for measuring macro-economic conditions for four representative regions of the world. Section 3 develops a framework for estimating regional financial risk factors based on Moody’s KMV expected default frequencies (EDFs), beyond the roles of credit risk ratings and macroeconomic factors. Section 4 provides empirical results on the estimation of a systemic risk index using the macroeconomic, financial risk and ratings factors. It examines the usefulness of these estimates for predicting systemic risk in the considered regions. Section 6 concludes.

2. The dynamic macroeconomic factor model

We formulate a state space model to estimate macroeconomic factors for different representative regions of the world using macroeconomic data observed at different frequencies. The model is based on an extended cumulator method that allows for autoregressive processes (for example, see Harvey (1990), Aruoba et al. (2009) and Sheen et al. (2013)). With this method, flow variables observed at a lower frequency are driven by the cumulated values of the unobserved state factors over the higher frequency observation period rather than by the state variables themselves. The cumulated state variables (or cumulators) are augmented to the state vector, with the low frequency flow variables loading on the cumulators and the high frequency variables loading on state factors.

We consider a five factor model—with four regional factors and one global factor. We model the world as composed of four representative regional types ($R = 4$): a rich super power—the United States ($US$); a super power bloc of countries—Europe ($EU$) comprising 27 countries; a typical rich small open economy—Australia ($AU$), that has had little experience of corporate

---

3Our paper is closely related to Schwaab et al. (2014). We differ in the following aspects: 1. Instead of using their more computational-demanding non-Gaussian state-space framework, we restrict ourselves to a linear-Gaussian framework to allow mixed frequency data. 2. Instead of using data on financial and non-financial firms, we focus on the former because we are interested in systemic financial crises. 3. We consider credit ratings important for measuring financial risk, and so we estimate and use a credit rating factor to help explain actual defaults. 4. They do not consider the zero inflation problem for defaults, and simply have a Binomial model. Instead we introduce a hurdle model. 5. We add China and Australia to our regional set outside the USA and the EU, while they use a differing mixture of other countries for defaults and macroeconomic data.

4In Appendix A we give an example of a mixed frequency model with one stock and one flow variable.
financial defaults in our sample period since 1990; and the rest of the emerging and developing world represented by China (CH), which also has experienced very few defaults.

We assume that the factors driving the macroeconomic conditions in each region can be decomposed into an unobserved global component that is common to all regions and an unobserved regional-specific component. The global factor component is assumed to follow an AR(1) process:

\[ M_{t+1}^W = \phi^W M_t^W + \sum_r \theta_r^W M_t^r + \eta_t^W, \]  

(2.1)

while the specific factor of region \( r \) is:

\[ M_{t+1}^r = \phi^r M_t^r + \theta_t^r M_t^W + \eta_t^r. \]  

(2.2)

where \( r = US, EU, AU, CH \).

A cumulator for a flow variable in region \( r \) can be expressed as:

\[ M_{t+1}^{r,c} = \psi_t^r M_t^{r,c} + M_t^r + \phi_t^r M_t^W + \eta_t^r. \]  

(2.3)

where \( \psi_t^r \) is an indicator variable, equal to 1 in periods \( t \) when low frequency variables are not observed and 0 otherwise.//

Observed variables used to estimate business cycle indices vary across studies. Output, consumption and investment data are used in Kose et al. (2003); employment, GDP and the term premium are used in Aruoba et al. (2009); the term premium, hours worked, a business confidence index, the terms of trade, the real exchange rate, GDP and job vacancies are used in Sheen et al. (2013); GDP, industrial production, unemployment rate, industrial confidence index, price data (inflation, stock market returns), the term premium and residential property prices are used in Schwaab et al. (2014). We use eight variables as detailed in Table 1. These variables cover a range of characteristics: stocks and flows; monthly, quarterly and annual data; and economic activity as well as pricing data in labour markets, product markets and asset markets.
Table 1: Variables used in the macroeconomic models

<table>
<thead>
<tr>
<th>Variables</th>
<th>Monthly</th>
<th>Quarterly</th>
<th>Annual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output ((gdp))</td>
<td>(US, EU, AU, CH)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation rate ((inf))</td>
<td>(US, EU, CH)</td>
<td>(AU)</td>
<td></td>
</tr>
<tr>
<td>Unemployment rate ((unr))</td>
<td>(US, EU, AU)</td>
<td></td>
<td>(CH)</td>
</tr>
<tr>
<td>Confidence index ((ci))</td>
<td>(US, EU)</td>
<td>(AU, CH)</td>
<td></td>
</tr>
<tr>
<td>Residential property price ((pp))</td>
<td>(US, EU, CH)</td>
<td></td>
<td>(AU)</td>
</tr>
<tr>
<td>Stock market return ((sr))</td>
<td>(US, EU, AU, CH)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job vacancies ((vac))</td>
<td>(US)</td>
<td>(EU, AU, CH)</td>
<td></td>
</tr>
<tr>
<td>Term of trade ((tot))</td>
<td>(US, CH)</td>
<td>(AU)</td>
<td>(EU)</td>
</tr>
<tr>
<td>Credit ((crd))</td>
<td></td>
<td></td>
<td>(US, EU, AU, CH)</td>
</tr>
<tr>
<td>Term premium ((tpm))</td>
<td>(US, EU, AU, CH)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Using the observed variables in Table 1, the observation equation of the state space model can be written as:

\[
\begin{bmatrix}
  y_{US\ gdp,t} \\
  \vdots \\
  y_{CH\ gdp,t} \\
  y_{US\ inf,t} \\
  \vdots \\
  y_{CH\ tpm,t}
\end{bmatrix}
= 
\begin{bmatrix}
  \gamma_{gdp}^{US} & 0 & \ldots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & \ldots & \ldots & \gamma_{tpm}^{CH}
\end{bmatrix}
\begin{bmatrix}
  y_{gdp,t-3}^{US} \\
  \vdots \\
  y_{inf,t-1}^{US} \\
  \vdots \\
  y_{tpm,t-1}^{CH}
\end{bmatrix}
+ 
\begin{bmatrix}
  0_{4 \times 5} \\
  \mu_{gdp}^{US} \\
  \mu_{inf}^{US} \\
  \vdots \\
  \mu_{tpm}^{CH}
\end{bmatrix}
\begin{bmatrix}
  M^W_t \\
  M_t^{US} \\
  \vdots \\
  M_t^{US,c} \\
  M_t^{CH} \\
  \vdots \\
  M_t^{CH,c}
\end{bmatrix}
+ 
\begin{bmatrix}
  \epsilon_{gdp,t}^{US} \\
  \vdots \\
  \epsilon_{inf,t}^{US} \\
  \epsilon_{tpm,t}^{CH}
\end{bmatrix}
\]

The transition equation for the state-space system with four regions is:

\[(2.4)\]
For model (2.4)-(2.5) to be identified, we fix the variance of factor innovations to unity.

2.1. Estimation results

We use data collected from Datastream for the period from January 1990 to December 2012. The time series for the observed variables are detrended using cubic polynomials and demeaned before estimation. GDP for CH and the real property prices of EU are deseasonalized using an ARIMA model as suggested by the NBER.

Estimates of the 136 parameters of the model (2.4)-(2.5) are given in Table B.5 in Appendix B. As expected, we find positive loadings on GDP (not significant for CH), inflation (not significant for US and AU), business confidence, real property prices, vacancies (not significant for EU). We find negative loadings on unemployment (not significant for CH) and the term premium (not significant for CH), picking up the stance of monetary policy. Regarding the transition parameters, $\theta$ and $\phi$, all factors exhibit high persistence. The world factor is positively driven by the lagged US factor, but negatively by the EU one, but the effects are small. Each region responds positively and significantly to the world factor. We find significant persistence of most individual data series and estimated error standard deviations.

Figure 1 presents the smoothed estimates of the world and regional macroeconomic factors, including their 95% confidence bands. The period from the early 1990s until the mid-2000s reflect what is widely known as the ‘Great Moderation’. The world factor is only significant (and negative) during the 2008-9 financial crisis. For the US, there are three significant periods: the positive build-up from 2004 to 2007, followed by a persistent collapse into the negative zone, with a good recovery after 2011. The EU is significant (and negative) only in the crisis, obviously buoyed by the performance of the German economy. The Australian economy only suffers significantly for a short time in the crisis. China exhibits a significant boom in the 1990s, stable (growth) in the 2000s, and just a minor and brief crisis in 2008-9.

---

5 See Geweke and Zhou (1996) for further details about identification conditions.
6 At the initial period of observation, the confidence interval is typically quite large, because a diffuse prior for the states is being used for the initial period. As soon as information becomes available, confidence shrinks to a reasonable interval.
3. The financial risk factor model

We now expand our monthly model to estimate the drivers of financial risk for the four representative regions \((R = 4)\). We assume that the default risk of bonds of individual financial companies in each region is driven by unobserved factors for regional financial risk, \(F_r\), after accounting for ratings, \(F_{Ar}\), and the estimated world and regional factors for macroeconomic conditions, \(M^W_t\) and \(M^r_t\).

Measuring financial distress for a region based on its historical defaults only is an almost impossible task, given the rather small number of historical default events. This is true in particular for three of our considered regions Europe, China and Australia, where less than 50 defaults in total could be observed during the time period from 1990 - 2012 in the financial sector. Thus, models based on actual defaults only will most likely not provide an informative picture of the actual situation with respect to financial distress. Instead we derive a complementary financial risk factor arising from expected default frequencies (EDFs) for a set of the largest financial companies in each region provided by Moody’s KMV, after accounting for their credit ratings. These EDFs are based on structural models of credit risk [Merton (1974)] that combine accounting based measures on debt with forward-looking information from equity markets. We believe these measures provide a timely and up-to-date picture of current stress in the financial sector, since they also take into account equity prices and volatility of financial institutions. Our derived financial risk and ratings factors will then used in the next section to help infer systemic risk for all regions, including those with few defaults.
We represent each region by a set of the eleven largest financial companies \((N = 11)\) among Moody’s rated company bonds in that region. Credit ratings and EDFs for these companies on a monthly frequency, obtained through Moody’s KMV CreditEdge over the period 1990:1-2012:12 \((T = 276)\), are used to estimate our financial risk factor model.

Since the EDF of company \(i\) in region \(r\) at date \(t\), \(EDF_{i,t}^r\), takes values between 0 and 1, we transform it using the logistic function into a real-valued variable \(z_{i,t}^r\) that is supported by a Normal distribution:

\[
z_{i,t}^r = \log \frac{EDF_{i,t}^r}{100 - EDF_{i,t}^r}, \quad i = 1...N, \quad t = 1...T, \quad r = 1...R.
\]

The transformed EDFs, \(z_{i,t}^r\), are assumed to depend on the Moody’s rating of company \(i\) in region \(r\) at date \(t\), the region of domicile of the company, and world and regional macroeconomic conditions:

\[
z_{i,t}^r = F_t^r D_i^r + F_t^A A_{i,t}^r + \kappa_t^W M_t^W + \kappa_t^r D_i^r M_t^r + \varepsilon_{i,t}^r, \quad (3.1)
\]

where \(D_i^r\) is an indicator variable for whether company \(i\) resides in region \(r\), \(A_{i,t}^r\) is the rating of company \(i\) in region \(r\), \(M_t^W\) is our estimated global macroeconomic index and \(M_t^r\) is our estimated macroeconomic index of region \(r\).

The observation equations can be expressed in matrix form as:

\[
\begin{bmatrix}
  z_{1,t}^{US} \\
  \vdots \\
  z_{N,t}^{CH}
\end{bmatrix} = 
\begin{bmatrix}
  1 & \ldots & 0 & A_{1,t}^{US} & \ldots & \ldots & \ldots & \ldots & 0 \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
  0 & \ldots & 1 & A_{N,t}^{CH}
\end{bmatrix} 
\begin{bmatrix}
  F_t^{US} \\
  \vdots \\
  F_t^{CH}
\end{bmatrix} + 
\begin{bmatrix}
  M_t^W \\
  \vdots \\
  M_t^{CH}
\end{bmatrix} 
\begin{bmatrix}
  \kappa_t^W \\
  \kappa_t^{US} \\
  \kappa_t^{CH}
\end{bmatrix} + 
\begin{bmatrix}
  \varepsilon_{1,t}^{US} \\
  \varepsilon_{N,t}^{US} \\
  \varepsilon_{p,t}^{CH}
\end{bmatrix}, \quad (3.2)
\]

We assume that the \(F\) factors evolve according to the transition equation:

\[
\begin{bmatrix}
  F_{t+1}^{US} \\
  \vdots \\
  F_{t+1}^{CH}
\end{bmatrix} = 
\begin{bmatrix}
  \beta_t^{US} & 0 & 0 & 0 & 0 \\
  \vdots & \vdots & \vdots & \vdots & \vdots \\
  0 & 0 & 0 & \beta_t^{CH} & 0 \\
  0 & 0 & 0 & 0 & \beta_t^{A}
\end{bmatrix} 
\begin{bmatrix}
  F_t^{US} \\
  \vdots \\
  F_t^{CH}
\end{bmatrix} + 
\begin{bmatrix}
  \eta_{t}^{US} \\
  \vdots \\
  \eta_{t}^{A}
\end{bmatrix}, \quad (3.3)
\]

where the vector \(\eta_t \sim N(0, Q)\), and \(Q\) is diagonal.

\(^7\)The model would not be identified if both the \(F\) factors and the \(\kappa\) factors were time-varying. Since the estimated macroeconomic conditions indices are time-varying, we choose to keep the \(\kappa\)s constant across time.
3.1. Key features of the EDF and the ratings data

For our EDF data from Moody’s, the eleven largest financial companies in the four regions that we use represent 59%, 27%, 100% and 81% of the total number of bond issues and 32%, 33%, 68% and 38% of the market capitalization in 2012 for the US, EU, AU and CH, respectively. In the case of China, which is a relatively new entrant to bond markets, we have also included financial companies from Hong Kong.

Bonds rather than companies are rated by Moody’s, and so multiple bond ratings may exist for a company on a single date. To obtain ratings for a company, we follow Fuertes and Kalotychou (2007) to use the lowest rating of senior unsecured bonds issued by that company. EDF data are provided monthly for the period 1990-2006 and then daily thereafter. We estimate the model using end-of-month data. Some companies may start their subscription to the ratings service later than others while some may cease earlier. For those that start later (or end early), we ignore their earlier (later) EDF data.

The distribution of observations across ratings are shown in Table 2. The data are more concentrated in higher ratings with about 90% of observations for Baal or better. However, movements of ratings to low levels may provide valuable information for the study of systemic risk. The conventional approach is to group ratings together and assign a separate factor to each group. In grouping low ratings with higher ratings, the information of sudden increases in risk may be lost. To preserve the information, we create a numerical ranking for assigned ratings, from 1 for the highest rating Aaa to 16 for the lowest rating B3. Thus movements to low ratings will increase the rank value. Since all ratings are considered together, a data sparsity issue does not arise.

---

8Since the Hong Kong and mainland Chinese economies are closely linked, our estimated CH macroeconomic factor is likely to have similar effects on the EDFs and defaults for financial companies in both.
Table 2: Observations across ratings

<table>
<thead>
<tr>
<th>Rating</th>
<th>Frequency</th>
<th>Count</th>
<th>Percent of Total Frequency</th>
<th>Cumulative Frequency</th>
<th>Count</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aa1</td>
<td>547</td>
<td>8.41</td>
<td>872</td>
<td>13.40</td>
<td>1607</td>
<td>25.81</td>
</tr>
<tr>
<td>Aa2</td>
<td>808</td>
<td>12.42</td>
<td>1680</td>
<td>25.81</td>
<td>2495</td>
<td>38.23</td>
</tr>
<tr>
<td>Aa3</td>
<td>1659</td>
<td>25.49</td>
<td>3339</td>
<td>51.31</td>
<td>5638</td>
<td>86.54</td>
</tr>
<tr>
<td>A1</td>
<td>858</td>
<td>13.18</td>
<td>4197</td>
<td>64.49</td>
<td>6038</td>
<td>92.79</td>
</tr>
<tr>
<td>A2</td>
<td>793</td>
<td>12.19</td>
<td>4990</td>
<td>76.67</td>
<td>6831</td>
<td>97.86</td>
</tr>
<tr>
<td>A3</td>
<td>587</td>
<td>9.02</td>
<td>5577</td>
<td>85.69</td>
<td>7418</td>
<td>96.95</td>
</tr>
<tr>
<td>Baa1</td>
<td>321</td>
<td>4.93</td>
<td>5898</td>
<td>90.63</td>
<td>8119</td>
<td>96.51</td>
</tr>
<tr>
<td>Baa2</td>
<td>63</td>
<td>0.97</td>
<td>5961</td>
<td>91.59</td>
<td>8800</td>
<td>98.33</td>
</tr>
<tr>
<td>Baa3</td>
<td>78</td>
<td>1.20</td>
<td>6039</td>
<td>92.79</td>
<td>9588</td>
<td>99.62</td>
</tr>
<tr>
<td>Ba1</td>
<td>118</td>
<td>1.81</td>
<td>6157</td>
<td>94.61</td>
<td>10735</td>
<td>99.46</td>
</tr>
<tr>
<td>Ba2</td>
<td>85</td>
<td>1.31</td>
<td>6242</td>
<td>95.91</td>
<td>11632</td>
<td>100.00</td>
</tr>
<tr>
<td>Ba3</td>
<td>157</td>
<td>2.41</td>
<td>6399</td>
<td>98.33</td>
<td>13211</td>
<td>100.00</td>
</tr>
<tr>
<td>B1</td>
<td>10</td>
<td>0.15</td>
<td>6409</td>
<td>98.48</td>
<td>13221</td>
<td>100.00</td>
</tr>
<tr>
<td>B2</td>
<td>24</td>
<td>0.37</td>
<td>6433</td>
<td>98.85</td>
<td>13245</td>
<td>100.00</td>
</tr>
<tr>
<td>B3</td>
<td>75</td>
<td>1.15</td>
<td>6508</td>
<td>100.00</td>
<td>13320</td>
<td>100.00</td>
</tr>
</tbody>
</table>

3.2. Estimation results for the financial risk model

The state space model in equations (3.2) and (3.3) has 59 parameters to be estimated: 44 in the diagonal measurement error covariance matrix $\Sigma$, 5 in the diagonal transition error covariance matrix $Q$, 5 in the diagonal matrix $\beta$ and 5 for the macroeconomic factors. The numerical optimization search using arbitrary starting values would be time-consuming. Since the model is equivalent to a stochastic regression model, we use the two-stage procedure proposed by Diebold and Li (2006) to obtain reasonable initial values of parameters. This reduces the computation time more than 6-fold.

The results are given in Table C.6 in Appendix C. The $\beta$ estimates show significantly high persistence of the five factors. The $\kappa$ estimates indicate that our estimated macro factors have no effect on the individual EDFs—however we will see in the next section that the regional macro factors play a crucial role in explaining systemic events. All estimated error standard deviations are significant.
The estimates for the ratings factor and the four regional financial risk factors are shown in Figure 2, including their 95% confidence intervals. The ratings factor measure declined in the 1990s, rose temporarily until 2005, fell marginally but not significantly prior to the crisis, but then rose significantly. This factor represents the generalized contribution of given ratings to the expected default risk of companies. If ratings do not change but EDFs are rising, the estimated ratings factor will increase to some extent. The significant rise seen in $F^A$ from 2007 to late 2009 was in the period when ratings agencies were criticised for failing to indicate adequately escalating risks. An important question answered in the next section is whether this ratings factor could predict systemic risk events. The US financial risk factor measured after accounting for ratings shows heightening risk from 1999 to 2002 (probably the ‘dot-com’ crisis), and a significant decline in apparent financial risk until 2007. After the 2008 crisis, this factor rose steeply and remained high until 2011, after which it recovered. The EU displays low relative financial risk until 2008, after which it begins to suffer from the US-induced crisis amplified later by its sovereign debt crisis. Australia exhibits low risk throughout, increasing marginally after 2010. China has almost no information on EDFs before 1996.
declining financial risk as its economy boomed until 2007, but a steep increase from 2008 as it financial system came under increasing pressure from its fast economic growth.

4. Modelling systemic indices

If a single major financial firm defaults or if a cluster of financial institutions defaults simultaneously, this will typically un settle confidence in the regional or even the global financial system. Therefore we define systemic risk as the time varying probability of sufficiently large defaults of one or more currently active financial institutions in a given economic region during a particular period. Such a definition of systemic risk or financial distress has been applied in several previous studies, including, for example, Giesecke and Kim (2011); Goodhart and Segoviano (2009); Schwaab et al. (2014). In particular we define a systemically relevant event as a default where the total market capitalization of the defaulting companies accounts for $k\%$ or more of a region’s market capitalization. We consider two values of $k$ to test whether we need to distinguish between a larger number of systemic events that also includes defaults of financial institutions with a lower regional market capitalization effect (i.e. a smaller $k$), and a smaller number of larger systemic events (i.e. a larger choice of $k$). These two cases are labelled ‘low hurdle’ and ‘high hurdle’ systemic events, though note that all ‘high hurdle’ events are contained in the set of ‘low hurdle’ events.

Information about financial defaults is extracted from Compustat and Moody’s Default Risk Service (DRS). Note that Compustat records defaulted companies when they file bankruptcy under Chapter 7 and Chapter 11, see, e.g., Duffie et al. (2007) for more details. These defaults may not necessarily cover all the cases of default listed in Moody’s DRS. This is due to the broader definition of default adopted by Moody’s DRS, including three types of credit events: (1) a missed or delayed disbursement of interest and/or principal; (2) bankruptcy, administration, legal receivership, or other legal blocks (perhaps by regulators) to the timely payment of interest and/or principal; or (3) a distressed exchange to help the borrower avoid default. We included both default events recorded in Moody’s DRS and Compustat, since both definitions of defaults are relevant for the systemic risk of the financial sector.

4.1. Features of the default data

Table 3: Observed defaults across geographical regions.

<table>
<thead>
<tr>
<th>Region</th>
<th>Frequency Count</th>
<th>Percent of Total Frequency</th>
<th>Cumulative Frequency Count</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>215</td>
<td>83.66</td>
<td>215</td>
<td>83.66</td>
</tr>
<tr>
<td>EU</td>
<td>39</td>
<td>15.18</td>
<td>254</td>
<td>98.83</td>
</tr>
<tr>
<td>AU</td>
<td>2</td>
<td>0.78</td>
<td>256</td>
<td>99.61</td>
</tr>
<tr>
<td>CH</td>
<td>1</td>
<td>0.39</td>
<td>257</td>
<td>100.00</td>
</tr>
</tbody>
</table>

9We use the market capitalization lagged by two years for defining these systemic events to avoid the impact of any default on the defaulted company’s market capitalization shortly prior to default. There can be several events occurring in a period if the period is sufficiently long. On each day of a month, we compute the number of defaults and these add to the month’s count if their total is not less than $k\%$ of the market.
As seen in Table ?, there were 257 defaults by listed financial companies in the four regions over the period 1990.1-2012.12, of which 83.66% occurred in the USA. Almost 90% of all defaults occurred since 2000.1. Severe default events occurred in the USA and the European Union in 2009. Only one occurred in Australia, in 2012. For China, they occurred in 2002-2003 and 2009-2010.

Figure 3 provides a plot of the severity of all observed default events measured by the percentage regional market capitalization of the defaulted financial institution. We find that only a relatively small number of defaults occurred to financial institutions with a market capitalization greater than \( k = 0.1\% \), the largest being the collapse of Lehman Brothers in 2008. Figure 4 displays the number of systemic events with a regional market capitalization of at least \( k\% \) as a function \( k \).

Figure 3: Severity of default events measured by the percentage regional market capitalization of the defaulted financial institution.
4.2. The hurdle model

Our monthly dataset of systemic financial default counts, $S^r_t$, for our 4 regions since 1990 contains a large number of zeroes. For this reason, we use a hurdle regression model to identify factors affecting and predicting such systemic events. In a hurdle model, a binomial probability model governs the binary outcome of whether the count variate (the number of systemic events) has a zero or a positive realization. If the realization is positive, the ‘hurdle’ is crossed and the conditional distribution of the positive realizations truncated at zero governs the number of counts.

The pooled hurdle model can be formally written as:

$$P(S^r_t = s^r) \sim f_{hurdle}(s^r | \pi^r_t, \lambda^r_t), \quad r = US, EU, AU, CH$$

$$f_{hurdle}(s^r | \pi^r_t, \lambda^r_t) = \begin{cases} 
1 - \pi^r_t & \text{if } s^r = 0 \\
\pi^r_t f_{TPoisson}(s^r | \lambda^r_t) & \text{if } s^r > 0 
\end{cases}$$

$$f_{TPoisson}(s^r | \lambda^r_t) = \frac{1}{1 - e^{-\lambda^r_t}} \left( \frac{\lambda^r_t}{s^r!} ight)^s e^{-\lambda^r_t}$$

where $S^r_t$ is the number of systemic events occurring in region $r$ in period $t$; $\pi^r_t$ is the probability that the number of systemic events is positive in region $r$ in period $t$ and $\lambda^r_t$ is the intensity parameter that controls the probability of systemic events in region $r$ in period $t$. Since the number of events in the Poisson process is always greater than or equal to 1, so is the expected number of events ($\frac{\lambda^r_t}{1 - e^{-\lambda^r_t}}$).

Since probabilities range between 0 and 1, we make the logit transformation of $\pi^r_t$ and assume this depends on $n$ covariates of region $r$. The ‘zero hurdle’ component of the model is:
\[
\begin{bmatrix}
\log \frac{\pi_t^{US}}{1 - \pi_t^{US}} \\
\vdots \\
\log \frac{\pi_t^{CH}}{1 - \pi_t^{CH}}
\end{bmatrix} = 
\begin{bmatrix}
1 & Z_{1,t}^{US} & Z_{2,t}^{US} & \ldots & Z_{n,t}^{US} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & Z_{1,t}^{CH} & Z_{2,t}^{CH} & \ldots & Z_{n,t}^{CH}
\end{bmatrix}
\begin{bmatrix}
\alpha_0 \\
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_n
\end{bmatrix}
\]

While for the ‘count’ component of the model, the logarithm of the Poisson parameter \( \lambda_t \) is assumed to depend on \( m \) covariates of region \( r \):

\[
\begin{bmatrix}
\log \lambda_t^{US} \\
\vdots \\
\log \lambda_t^{CH}
\end{bmatrix} = 
\begin{bmatrix}
1 & X_{1,t}^{US} & X_{2,t}^{US} & \ldots & X_{m,t}^{US} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & X_{1,t}^{CH} & X_{2,t}^{CH} & \ldots & X_{m,t}^{CH}
\end{bmatrix}
\begin{bmatrix}
\delta_0 \\
\delta_1 \\
\delta_2 \\
\vdots \\
\delta_m
\end{bmatrix}
\]

As mentioned above, we distinguish between two different cases based on the choice of \( k \): the case where also ‘low hurdle’ systemic events are measured (\( k = 0.002\% \)), i.e. a larger number of defaults is included in the sample, such that we have a total of 95 systemic events for all regions. When only ‘high hurdle’ systemically relevant defaults are considered (\( k = 0.01\% \)), we obtain 49 systemic events for all regions, i.e the sample contains a smaller number of more severe default events.

The set of covariates tried in model estimation include the derived regional and world macroeconomic factors, the regional financial risk factor, the ratings factor, a credit default spread index (cds), spreads between interest rates on interbank loans and on short-term government debt (spread), changes in term premium (dtpm), changes in 3-month Treasury bill rate (dm3), the excess return of property price over stock market return (epp), the stock market return (sr), the volatility of stock market indices (vol), and their lags.\(^{10}\)

4.3. Hurdle model estimates

Table 4 gives the results for the hurdle model. To reach these estimates, we began with the most general set of covariates and sequentially eliminated the most insignificant ones.

\(^{10}\)The volatility of stock market indices is estimated using an exponentially weighted moving average (EWMA) model with \( \lambda = 0.94 \). The data required to calculate liquidity spreads are difficult to obtain, especially for China. Instead, we use the US TED spread for all regions since several studies have found that the TED spread drives systemic risk not only in the USA but also in other countries. All explanatory variables are demeaned and scaled to have unit variance so that their impacts on systemic risk can be compared.
### Table 4: Hurdle Model Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>Low hurdle</th>
<th>High hurdle</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Count component</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\delta_0)</td>
<td>-1.9</td>
<td>-2.82</td>
</tr>
<tr>
<td>(\delta_{MR-5})</td>
<td>-0.88</td>
<td>-1.16</td>
</tr>
<tr>
<td><strong>Zero component</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha_0)</td>
<td>-2.97</td>
<td>-4.13</td>
</tr>
<tr>
<td>(\alpha_{FRt})</td>
<td>0.38</td>
<td>0.74</td>
</tr>
<tr>
<td>(\alpha_{MRt})</td>
<td>-0.28</td>
<td></td>
</tr>
<tr>
<td>(\alpha_{MRt-3})</td>
<td>-0.26</td>
<td></td>
</tr>
<tr>
<td>(\alpha_{FA})</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>(\alpha_{FA-1})</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td>(\alpha_{tedt})</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td><strong>Pseudo (R^2)</strong></td>
<td>0.09</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Note: Significance at the 1%, 5% and 10% level is denoted by \(***\), \(**\) and \(*\) respectively.

The top two rows provide the estimates for the ‘count’ component of the hurdle model, given that systemic events occur. Interestingly, the only statistically significant parameters are the intercept and the regional macroeconomic factor lagged five months. This result holds in both cases—low and high hurdle systemic events. Thus the intensity of a systemic crisis in a region depends negatively on the macroeconomic state. The better the state, the fewer the number of systemic events.

The bottom rows show the estimates for the ‘zero’ component of the hurdle. The probability of a systemic event depends positively and significantly on the regional financial risk factors and on the general ratings factor, negatively and significantly on the regional macroeconomic factors.

#### 4.3.1. Comparing low and high hurdle systemic crises

There are some differences between measuring low and high hurdle defaults in the financial sector. As seen in Table 4, the latter has a positively significant TED spread effect, suggesting that larger systemic events are closely associated with (but not necessarily caused by) global credit market crises. We also find that a one month lag of the regional macro factors and the ratings factor are instead preferred to explain major defaults. This indicates that these covariates are likely to be good predictors for larger systemic events. For larger systemic events, ratings play a statistically bigger role. In general, the absolute values of estimated parameters

---

\(^{11}\)Since most of the defaults took place after 2000, we re-estimated the hurdle model for the shorter period 2001.1 to 2012.12. The results were similar, except for the fact that the ratings factor no longer plays a statistical role. This suggests that the ratings became less informative when the intensity of systemic events increased.
are greater for high hurdle events, while the regional macro factor is marginally less significant for major defaults in the financial sector. The (McFadden) pseudo $R^2$ is larger for the high hurdle case, but this statistic cannot be used to compare estimations with different datasets.

For each region and crisis case, we present in Figures 5 to 12 the actual number of systemic events, the estimated probability of a systemic event ($\pi_t^r$) and the estimated number of systemic events ($\pi_t^r \lambda_t^r / (1 - e^{-\lambda_t^r})$). Conditional on the number of systemic events being positive, $\lambda_t^r / (1 - e^{-\lambda_t^r})$ is the expected number of systemic events, which must be greater than or equal to 1. The expected number of systemic events ($\pi_t^r \lambda_t^r / (1 - e^{-\lambda_t^r})$ is thus always larger than the probability of a systemic event ($\pi_t^r$) and the larger the difference, the more likely that multiple systemic events will occur in the period. For low hurdle crises that have many more events than high hurdle crises, the difference can be and is noticeable.

![Figure 5: Systemic index for the USA—including low hurdle systemic events, i.e. all defaults of financial institutions with regional market capitalization of at least $k = 0.002\%$.](image-url)
Figure 6: Systemic index for the USA—high hurdle systemic events, i.e. defaults of financial institutions with regional market capitalization of at least $k = 0.01\%$.

The two cases in the US have some important similarities and differences. Generally, the probability of a smaller systemic event is higher than of a larger one. The probability of observing a low hurdle and a high hurdle systemic event increased dramatically in 2008 to 25% and 30%, respectively. The probability of a high hurdle event declined to almost zero by the end of 2012, but was about 3% for a low hurdle one.

Figure 7: Systemic index for the European Union—low hurdle systemic events, i.e. all defaults of financial institutions with regional market capitalization of at least $k = 0.002\%$. 
Figure 8: Systemic index for the European Union—high hurdle systemic events, i.e. defaults of financial institutions with regional market capitalization of at least $k = 0.01\%$.

The EU differs noticeably from the US in the period after 2008, when the probability of a systemic event begins a second escalation from 2011. The probability of low hurdle (high) systemic events reached 19% (16%) at the end of 2012.

Figure 9: Systemic index for Australia—low hurdle systemic events, i.e. all defaults of financial institutions with regional market capitalization of at least $k = 0.002\%$. 

21
Figure 10: Systemic index for Australia—high hurdle systemic events, i.e. defaults of financial institutions with regional market capitalization of at least $k = 0.01\%$.

For Australia, which had only two low hurdle systemic events, the inferences have to come largely from the default experiences in the US and EU. These suggest that the probability of a low hurdle or high hurdle event was as high in 2008 as in the US. When a low hurdle event occurred in 2012, the probability was 10%. However the probability of a high hurdle systemically relevant event was very small.

Figure 11: Systemic index for China—low hurdle systemic events, i.e. all defaults of financial institutions with regional market capitalization of at least $k = 0.002\%$. 

22
China experienced just one default in 2009. Again based on inference from the US and EU, the probability of a low hurdle or high hurdle crisis rose dramatically in 2008 to reach 12% and 15% respectively. It continued to have a heightened risk of a low hurdle systemic crisis through 2012.

A key question is whether our model would have predicted the existence and severity of the 2008 financial crisis. Since regional macroeconomic factors play a significant role over the whole sample for explaining the existence and severity of systemic events, they ought to have had predictive power for the crisis. Looking back at Figure 1, the US regional macroeconomic factor turned down quite significantly in late 2007, well before the EU, Australia and China factors. House prices had begun to fall in the US, which affected business confidence. Both of these variables had significant estimated loadings for the US macroeconomic factor, which in turn had a significant impact on the probability and intensity of a systemic event in mid-2008.

5. Conclusions

Our top-down approach to estimating systemic risk across four representative regions of the world has established the importance of regional macroeconomic factors for predicting the likelihood and intensity of a systemic event. The likelihood of an event depends also on regional financial risk and ratings factors. We construct the unobserved macroeconomic factors using a mixed-frequency state-space model, employing a wide range of relevant macroeconomic variables. The financial risk and ratings factors arise from a state-space model of expected financial defaults of the eleven largest financial institutions in each region.

Our model could have predicted the 2008 crisis through the rise in the systemic probability in late 2007. This was on account of the significant negative effects on the US macroeconomic factor value of the fall in US house prices and business confidence in late 2007, which in turn
significantly increased the probability and intensity of a systemic event in mid-2008.

Stronger macroeconomic conditions will reduce the probability and intensity of a systemic event, both low hurdle and high hurdle ones. Policymakers need to ensure macroeconomic stability over the longer term to avoid any systemic crises. Ratings are relevant for predicting systemic events, but are insufficient. Just prior to the 2008 crisis, that insufficiency was pronounced. After accounting for ratings, the residual financial risk in a region arising out of Moody’s KMV expected default frequency data is an important predictor of the probability of a systemic event. Therefore, financial regulators and supervisors need to ensure that these financial risk factors are not escalating. Credit market dysfunction represented by the TED spread is closely associated with high hurdle systemic financial crises. Therefore this indicator is a useful bellwether of a major systemic event.
Appendix A. An example of a small mixed-frequency state-space model

Consider a single factor monthly model of two observed time series where the first series $y_{1,t}$ is a stock variable or a flow variable observed at monthly frequency and the second series $y_{2,t}$ is a flow variable observed at quarterly frequency. Assume the transition equation for the macroeconomic factor $M_t$ at a monthly frequency is:

$$M_{t+1} = \phi M_t + \eta_t.$$  

The first monthly variable follows an idiosyncratic AR(1) process and loads on factor $M_t$:

$$y_{1,t} = \gamma_1 y_{1,t-1} + \mu_1 M_t + \epsilon_{1,t},$$

while the quarterly observed flow variable, $y_{2,t}$, has a quarterly AR(1) process but loads on the cumulated aggregate $M^c_t$ of factor $M_t$:

$$y_{2,t} = \gamma_2 y_{2,t-3} + \mu_2 M^c_t + \epsilon_{2,t}$$

where the monthly $M^c_t$ is defined for $t = 3(q-1) + m$ as:

$$M^c_{3(q-1)+m} = \sum_{s=1}^{m} M_{3(q-1)+s}, q = 1, 2, ..., Q, m = 1, 2, 3$$

and $Q$ is the number of quarters observed, and $m$ indexes the month within a quarter. The process for the cumulator $M^c_t$ can be expressed more succinctly as:

$$M^c_{t+1} = \psi_{2,t} M^c_t + M_{t+1} = \psi_{2,t} M^c_t + \phi M_t + \eta_t,$$  \hspace{1cm} (A.1)

where $\psi_{2,t}$ is an indicator variable, equal to 1 in periods $t$ when $y_{2,t}$ is not observed and 0 otherwise.

The mixed-frequency state-space model becomes:

$$
\begin{bmatrix}
    y_{1,t} \\
    y_{2,t}
\end{bmatrix} =
\begin{bmatrix}
    \gamma_1 & 0 \\
    0 & \gamma_2
\end{bmatrix}
\begin{bmatrix}
    y_{1,t-1} \\
    y_{2,t-3}
\end{bmatrix} +
\begin{bmatrix}
    \mu_1 & 0 \\
    0 & \mu_2
\end{bmatrix}
\begin{bmatrix}
    M_t \\
    M^c_t
\end{bmatrix} +
\begin{bmatrix}
    \epsilon_{1,t} \\
    \epsilon_{2,t}
\end{bmatrix}
$$  \hspace{1cm} (A.2)

$$
\begin{bmatrix}
    M_{t+1} \\
    M^c_{t+1}
\end{bmatrix} =
\begin{bmatrix}
    \phi & 0 \\
    0 & \psi_{2,t}
\end{bmatrix}
\begin{bmatrix}
    M_t \\
    M^c_t
\end{bmatrix} +
\begin{bmatrix}
    \eta_t \\
    1
\end{bmatrix}
$$  \hspace{1cm} (A.3)

Appendix B. Macroeconomic factor model estimates
Table B.5: Estimated parameters of macroeconomic model

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\theta &amp; \phi$</th>
<th>$\gamma$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{US}$</td>
<td>0.01 ***</td>
<td>$\phi_{US}^{W}$</td>
<td>0.70 ***</td>
</tr>
<tr>
<td>$\mu_{inj}$</td>
<td>0.01</td>
<td>$\phi_{US}^{W}$</td>
<td>0.04 *</td>
</tr>
<tr>
<td>$\mu_{uu}$</td>
<td>-0.01 ***</td>
<td>$\phi_{US}^{W}$</td>
<td>-0.06 **</td>
</tr>
<tr>
<td>$\mu_{cb}$</td>
<td>0.01 **</td>
<td>$\phi_{US}^{W}$</td>
<td>0.03</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.02 ***</td>
<td>$\phi_{US}^{W}$</td>
<td>-0.04</td>
</tr>
<tr>
<td>$\mu_{vac}$</td>
<td>0.01 **</td>
<td>$\phi_{US}^{W}$</td>
<td>0.97 ***</td>
</tr>
<tr>
<td>$\mu_{tot}$</td>
<td>-0.00</td>
<td>$\phi_{EU}^{W}$</td>
<td>0.99 ***</td>
</tr>
<tr>
<td>$\mu_{cd}$</td>
<td>-0.01</td>
<td>$\phi_{EU}^{W}$</td>
<td>0.94 ***</td>
</tr>
<tr>
<td>$\mu_{sr}$</td>
<td>0.00</td>
<td>$\phi_{CH}^{W}$</td>
<td>0.82 ***</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>-0.01</td>
<td>$\phi_{US}^{W}$</td>
<td>0.14</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.01 ***</td>
<td>$\phi_{EU}^{W}$</td>
<td>1.13 **</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.01 ***</td>
<td>$\phi_{EU}^{W}$</td>
<td>0.86 *</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>-0.01 ***</td>
<td>$\phi_{EU}^{W}$</td>
<td>0.34 ***</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.03 ***</td>
<td>$\phi_{EU}^{W}$</td>
<td>0.84 ***</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.03 ***</td>
<td>$\phi_{EU}^{W}$</td>
<td>0.91 ***</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.03</td>
<td>$\phi_{EU}^{W}$</td>
<td>0.36</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>-0.01</td>
<td>$\phi_{EU}^{W}$</td>
<td>0.35 *</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.02</td>
<td>$\phi_{EU}^{W}$</td>
<td>0.93 ***</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.00</td>
<td>$\phi_{EU}^{W}$</td>
<td>0.00</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>-0.01 ***</td>
<td>$\phi_{EU}^{W}$</td>
<td>1.00 ***</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.01</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.01</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>-0.01</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.10</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.04</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.02</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.02</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.06</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.00</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>-0.01</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.00</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.18 ***</td>
<td>$\gamma_{pp}$</td>
<td>0.06</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>-0.01</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.17 ***</td>
<td>$\gamma_{pp}$</td>
<td>0.83 ***</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.04</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.15</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>-0.08</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.11</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.03</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
<tr>
<td>$\mu_{pp}$</td>
<td>0.03</td>
<td>*</td>
<td>$\gamma_{pp}$</td>
</tr>
</tbody>
</table>

Note: Significance at the 1%, 5% and 10% level is denoted by "***", "**", and ", respectively.
Appendix C. Financial risk model estimates

Table C.6: Estimated parameters of financial risk model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta^A )</td>
<td>1.00 ***</td>
<td>( \sigma_{\epsilon,1} )</td>
<td>0.51 ***</td>
<td>( \sigma_{\epsilon,24} )</td>
<td>0.87 ***</td>
</tr>
<tr>
<td>( \beta^{US} )</td>
<td>0.99 ***</td>
<td>( \sigma_{\epsilon,2} )</td>
<td>1.43 ***</td>
<td>( \sigma_{\epsilon,25} )</td>
<td>0.09 ***</td>
</tr>
<tr>
<td>( \beta^{EU} )</td>
<td>0.99 ***</td>
<td>( \sigma_{\epsilon,3} )</td>
<td>1.72 ***</td>
<td>( \sigma_{\epsilon,26} )</td>
<td>0.81 ***</td>
</tr>
<tr>
<td>( \beta^{AU} )</td>
<td>0.97 ***</td>
<td>( \sigma_{\epsilon,4} )</td>
<td>0.45 ***</td>
<td>( \sigma_{\epsilon,27} )</td>
<td>1.41 ***</td>
</tr>
<tr>
<td>( \beta^{CH} )</td>
<td>0.99 ***</td>
<td>( \sigma_{\epsilon,5} )</td>
<td>1.63 ***</td>
<td>( \sigma_{\epsilon,28} )</td>
<td>1.72 ***</td>
</tr>
<tr>
<td>( \kappa^W )</td>
<td>5.94</td>
<td>( \sigma_{\epsilon,6} )</td>
<td>1.97 ***</td>
<td>( \sigma_{\epsilon,29} )</td>
<td>1.05 ***</td>
</tr>
<tr>
<td>( \kappa^{US} )</td>
<td>1.60</td>
<td>( \sigma_{\epsilon,7} )</td>
<td>2.05 ***</td>
<td>( \sigma_{\epsilon,30} )</td>
<td>1.72 ***</td>
</tr>
<tr>
<td>( \kappa^{EU} )</td>
<td>0.92</td>
<td>( \sigma_{\epsilon,8} )</td>
<td>0.66 ***</td>
<td>( \sigma_{\epsilon,31} )</td>
<td>0.41 ***</td>
</tr>
<tr>
<td>( \kappa^{AU} )</td>
<td>1.84</td>
<td>( \sigma_{\epsilon,9} )</td>
<td>0.46 ***</td>
<td>( \sigma_{\epsilon,32} )</td>
<td>0.89 ***</td>
</tr>
<tr>
<td>( \kappa^{CH} )</td>
<td>0.35</td>
<td>( \sigma_{\epsilon,10} )</td>
<td>1.03 ***</td>
<td>( \sigma_{\epsilon,33} )</td>
<td>1.18 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,11} )</td>
<td>0.14 ***</td>
<td>( \sigma_{\epsilon,12} )</td>
<td>2.27 ***</td>
<td>( \sigma_{\epsilon,34} )</td>
<td>0.52 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,13} )</td>
<td>0.07 *</td>
<td>( \sigma_{\epsilon,14} )</td>
<td>1.53 ***</td>
<td>( \sigma_{\epsilon,35} )</td>
<td>0.74 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,15} )</td>
<td>0.78 ***</td>
<td>( \sigma_{\epsilon,16} )</td>
<td>0.26 ***</td>
<td>( \sigma_{\epsilon,36} )</td>
<td>1.98 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,17} )</td>
<td>0.38 ***</td>
<td>( \sigma_{\epsilon,18} )</td>
<td>0.67 ***</td>
<td>( \sigma_{\epsilon,37} )</td>
<td>0.72 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,19} )</td>
<td>0.35 ***</td>
<td>( \sigma_{\epsilon,20} )</td>
<td>1.24 ***</td>
<td>( \sigma_{\epsilon,38} )</td>
<td>0.25 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,21} )</td>
<td>2.10 ***</td>
<td>( \sigma_{\epsilon,22} )</td>
<td>0.71 ***</td>
<td>( \sigma_{\epsilon,39} )</td>
<td>1.11 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,23} )</td>
<td>1.00 ***</td>
<td>( \sigma_{\epsilon,24} )</td>
<td>0.87 ***</td>
<td>( \sigma_{\epsilon,40} )</td>
<td>0.76 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,25} )</td>
<td>0.09 ***</td>
<td>( \sigma_{\epsilon,26} )</td>
<td>0.81 ***</td>
<td>( \sigma_{\epsilon,41} )</td>
<td>0.82 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,27} )</td>
<td>1.41 ***</td>
<td>( \sigma_{\epsilon,28} )</td>
<td>1.72 ***</td>
<td>( \sigma_{\epsilon,42} )</td>
<td>0.55 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,29} )</td>
<td>1.05 ***</td>
<td>( \sigma_{\epsilon,30} )</td>
<td>1.72 ***</td>
<td>( \sigma_{\epsilon,43} )</td>
<td>0.01 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,31} )</td>
<td>0.41 ***</td>
<td>( \sigma_{\epsilon,32} )</td>
<td>0.89 ***</td>
<td>( \sigma_{\epsilon,44} )</td>
<td>0.17 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,33} )</td>
<td>1.18 ***</td>
<td>( \sigma_{\epsilon,34} )</td>
<td>0.52 ***</td>
<td>( \sigma_{\epsilon,45} )</td>
<td>0.11 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,35} )</td>
<td>0.74 ***</td>
<td>( \sigma_{\epsilon,36} )</td>
<td>1.98 ***</td>
<td>( \sigma_{\epsilon,46} )</td>
<td>0.12 ***</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,37} )</td>
<td>0.72 ***</td>
<td>( \sigma_{\epsilon,38} )</td>
<td>0.25 ***</td>
<td>( \sigma_{\epsilon,47} )</td>
<td>0.21 ***</td>
</tr>
</tbody>
</table>

Note: Significance at the 1%, 5% and 10% level is denoted by ***, **, and * respectively
References


