Welfare Transfers and Intra-Household Trickle-Down: A Model with Evidence from the US Food Stamp Program

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ABSTRACT

We examine the case for maintaining welfare and income redistribution programs even when their adverse general equilibrium effects reduce total earnings of poor households. Using a Cournot model of intra-household decision-making, we show that even if welfare cutbacks generate large increases in household income, these may still reduce the well-being of children and elderly dependants. Our model also explains the higher marginal propensity to consume food out of food stamps in the US, compared to that out of market income, noted in earlier empirical studies. We find evidence consistent with our argument in data from a US Food Stamp experiment.

JEL Classification H23, H31, I38, J16

Keywords: Food Stamp Program, Welfare Transfers, Cash-out Puzzle, Cournot Model, Intra-household Distribution, Engel Curves
1. Introduction

Governments often spend a large part of their budgets on welfare, anti-poverty and income redistribution programs. In recent decades, many of these programs have been subjected to strong criticism, and have been restricted or rolled back. Reductions in welfare and redistributive expenditures typically form a key component of structural adjustment/stabilization strategies in developing economies. ‘Austerity measures’ of this kind have been posited as necessary for developed economies as well. The main argument offered in justification has usually run along the following lines. Disincentive effects, on the non-poor, of taxes and state borrowings used to fund welfare programs reduce private initiative and investment, and thereby, employment and growth. Consequently, market earnings of the poor are lower than what they would otherwise be. Furthermore, a significant part of the spending on these programs actually represents bureaucratic waste and corruption.\(^1\) In effect, therefore, welfare income largely ends up ‘crowding out’ market income for poor households, on a more than one to one basis. General equilibrium effects of welfare cutbacks will generate additional market income for erstwhile beneficiaries, to an extent that will more than compensate for their loss of welfare income. By increasing their total income, such cutbacks will improve the well being of the poor.

Whether a roll-back of welfare measures actually has (or indeed can have) as robustly positive an impact on market earnings of poor households as envisaged in the view just outlined, is an issue that has received much critical attention.\(^2\) This paper provides a different kind of caveat. We show that cutbacks in welfare and income redistribution programs may generate large income gains for poor households, yet nevertheless reduce the welfare of dependent and economically inactive members, viz., children and the elderly. Sacrificing rigor for clarity, one may put the matter thus. Standard justifications of welfare programs argue that prosperity for the rich need not ‘trickle down’ to the poor. We develop an alternative justification by arguing instead that parental prosperity within poor households need neither ‘trickle down’ to children, nor, indeed, ‘trickle up’ to grandparents.

Many welfare programs involve in-kind transfers with rationing. A major example is that of free/subsidized provision of food, either through food stamps (as in the US and Sri Lanka), or through public distribution systems (as in many developing economies). Other common examples include the provision of school lunches, uniforms and educational material, of feeding programs for pre-schoolers, of basic medical support for pre-teens, pregnant and lactating mothers and the elderly, of housing, etc. Cash pensions to the elderly can also be thought of, for analytical purposes, as in-kind transfers (of the composite good consumed by the elderly) made by the state to their adult progeny. Dependent members of the household, i.e., children and the elderly, usually benefit from larger household expenditure on

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1 Okun (1975) once described such programs as carrying money from the rich to the poor in a ‘leaky bucket’.

2 For recent overviews, see Lipton and Ravallion (1995) and van de Walle and Neade (1995).
commodities typically provided through in-kind transfers. The link is direct for items that can only be consumed by them, and for domestic public goods such as housing and certain types of health inputs. Their nutritional levels are also likely to improve with increased food availability within the household. Thus, *if* a market ‘cash-out’, i.e., a replacement of state provision of commodities which benefit children and the elderly by additional market income for economically active members of the household, led to lower spending on these commodities, then the former would be worse off. In particular, reduced spending on children’s commodities may, by reducing their accumulation of human capital, make it more difficult for them to access market opportunities in the future. Consequently, such a growth driven process of poverty reduction may, paradoxically, generate a poverty trap and thereby become unsustainable in the long run.

The standard literature on in-kind transfers however allows this possibility only when households are *constrained* (i.e., they receive, in kind, an amount greater than their desired consumption). This literature typically follows the ‘unitary’, or ‘income-pooling’, approach (systematized largely by Becker (1965, 1981)) to the modeling of household consumption behavior, whereby a household is assumed to behave as a single individual. Within this framework, an exact market cash-out should not make any difference to the consumption behavior of unconstrained households (i.e., households which additionally purchase, from the open market, a positive amount of the commodity transferred). Hence, a more than exact cash-out, which increases total household income, should actually *increase* household purchase of these commodities (assuming they are normal goods). Empirical studies often find the proportion of constrained households to be quite small. Thus, under the standard framework, granted a more than exact compensating increase in household market income, the practical importance of our caveat against welfare cutbacks is likely to be minor.

This conclusion however changes drastically if grounds exist for taking the following claims, running contrary to the prediction of the standard analysis, seriously. First, market cash-out of in-kind transfers of items that benefit children and the elderly is likely to lower household consumption of these items, even when households are unconstrained. Second, household spending on children and the elderly may go down even when cutbacks in cash welfare transfers are more than compensated by additional market income. Examining such grounds is the primary motivation for our paper.

Prior justification for taking the first possibility seriously has been generated by empirical studies of the US Food Stamp Program. This research has brought to light the puzzling fact that the marginal propensity to consume food out of food stamps seems to be much (three to ten times) higher

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3 Assuming, of course, a lump-sum increase in market income, and thereby abstracting from possible substitution effects that increased returns to market participation may generate. See, for example, Rosenzweig (1986). Throughout this paper we shall focus on the relative income effects of market and welfare earnings, and abstract from possible substitution effects.

4 Studies of the US Food Stamps Program have typically found only 5-15% of the beneficiary households to be constrained (Fraker (1990)).
than that out of cash income, even for unconstrained households.\(^5\) This would seem to imply that a substitution of food stamps by market income would significantly reduce household food purchase, thereby jeopardizing nutritional security of children and the elderly. In developing our general theoretical justification for welfare transfers, we also explain this, so-called, ‘cash-out puzzle’.

We explore the following hypothesis. Once the household is modeled in a ‘collective’ (Alderman et al (1995)) fashion, by explicitly formulating household decisions as the outcome of the interaction between individual members with possibly different preferences and endowments, welfare and market incomes will turn out to have very different consequences for household consumption.

This shift in modeling strategy immediately opens up a simple intuitive argument along the following lines. Consider a household consisting of two income earners, M and F, children and dependent parents. When the household receives, say, $100 worth of food via food stamps, M spends all his cash income on non-food items, while F spends, say, $25 out of her own cash income on food. Thus, the household as a unit is unconstrained. Now suppose, instead, the household receives no food stamps, but M’s own cash income goes up by $150. M however chooses to spend none of his incremental income on food. Then, even if F increases her spending on food in response, total household food purchase may fall. This will lower the amount provided to children and the elderly, thereby reducing their well being.

Passing conjectures along similar lines in the context of the US Food Stamp Program have been made earlier, for example by Alderman et al (1997, p. 278), Wilde and Ranney (1996) and Senauer and Young (1986). Yet, such conjectures have not been subjected to empirical scrutiny. Our intuitive reasoning, if valid, would imply that welfare income has a different impact on household consumption than market income when households consist of more than one independent decision-maker. However, demand behavior of single-adult households, reflecting the preferences and endowments of a single decision-maker, should follow the prediction of the standard, unitary, model. Hence, the cash-out puzzle would be generated primarily by the demand behavior of multi-adult households. On the other hand, if factors other than intra-household allocation processes play a major role in generating the puzzle, then single-adult households should exhibit the puzzle to a significant extent as well. The empirical innovation of this paper lies in analyzing the demand behavior of single and multiple adult households separately. Our specific empirical contribution consists of evidence, presented in Section 2, that the cash-out puzzle may indeed be driven primarily by intra-household allocation processes, rather than by some other factor. Analyzing data generated by experimental cashing out of food stamps in San Diego county, California, we find no evidence of any cash-out puzzle for single-adult headed households. Multi-adult households however exhibit marginal propensity to consume food out of food stamps much larger than that out of market income.

\(^5\) See the seventeen studies reviewed in Fraker (1990).
In the theoretical literature on intra-household allocation, modeled in terms of games involving voluntary contributions to domestic public goods, it is well known that a lump-sum income redistribution from contributory to non-contributory agents will, typically, reduce the amount of the domestic public good supplied in equilibrium. The existing literature, however, does not provide a rigorous examination of the conditions under which households will necessarily contain a non-contributory agent. Nor does it provide an explicit analysis of the case where the income redistribution comes about as an indirect consequence of market cash-out of welfare payments, and is associated with an increase in total household income. Our specific theoretical contribution lies in providing an integrated analysis of intra-household allocation which simultaneously addresses these two issues.

This we do in Section 3. We model interaction within a household containing multiple income earners in Cournot fashion, where household consumption of some commodity provided through in-kind transfers has the formal property of being a domestic public good. Funding of welfare transfers is modeled in terms of a tax imposed on market income, which generates a positive, possibly large, deadweight loss. We show that, under quite reasonable restrictions on preferences and individual market earnings, our model predicts the following. There must necessarily exist an interval of welfare transfers, within which one agent will choose not to spend any cash on the domestic public good, even though the household as a whole is unconstrained. Within this range, cutbacks in welfare transfers, even if more than compensated by additional market income, must nevertheless reduce household consumption of the domestic public good, thereby reducing the welfare of dependants.

Our result can be extended, though under somewhat stronger conditions, to the case where welfare payments are made in cash. Furthermore, they can be interpreted as justifying the payment of transfers in kind rather than in cash. These generalizations are discussed in Section 4. We summarize and conclude in Section 5. All proofs are relegated to the Appendix.

2. Evidence

As mentioned earlier, empirical research has consistently found the marginal propensity to consume food out of food stamps in the US to be much higher than that out of cash income for unconstrained households (those who receive food stamps, but spend positive amounts of cash income on food). Without making any firm conclusions, Senauer and Young (1986) suggest three possible explanations for the cash-out puzzle other than intra-household distribution effects. These are: (a)

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7 For example, Dasgupta (2001) explicitly assumes that both spouses contribute to the domestic public good. Carter and Katz (1997) present a ‘conjugal contract’ model where the domestic public good is produced by means of voluntarily contributed domestic labor, whereas our focus is on monetary contributions and welfare transfers. Both Lundberg and Pollack (1993) and Chen and Woolley (2001) focus on general properties of the non-cooperative model of intra-household allocation, rather than the specific issue of non-contribution and welfare transfers.
budgeting effects: recipient households use the stamp amount as a signal about how much to spend on food (and perhaps feel a sense of gratitude that causes them to relocate towards higher levels of food consumption); (b) consumption smoothing: if food stamps constitute a more reliable source of income than other sources, the permanent income hypothesis would predict that propensities to consume out of food stamps would also be higher, and (c) lumpy purchase: stamp beneficiaries take fewer trips to the store. To these conjectures, Levedahl (1995) adds that of marginal welfare stigma.\(^8\) If it is indeed the case that reasons such as these, rather than intrahousehold distribution effects, are the primary driving force behind the cash-out puzzle, then we would expect both single-adult households and multi-adult households to exhibit the puzzle. However, if intrahousehold distribution effects are the dominant factor, then single-adult households should not exhibit the puzzle to any significant extent. Thus, if we find evidence that the cash-out puzzle is being driven primarily by the consumption behavior of multi-adult households, then this would provide empirical grounds for taking the intra-household distribution explanation seriously. We present such evidence in this section.

We consider data from a ‘cash-out’ experiment conducted by the Food and Consumer Service of the United States Department of Agriculture in the late 1980s where food stamp participants were given cash instead of the traditional coupons. Other researchers such as Levedahl (1995) and Fraker, Martini and Ohls (1995) have found evidence of the cash-out puzzle in this data-set. We analyze the same data-set separately for single and multi-adult households in order to show that their finding can be largely explained by the consumption behavior of multi-adult, rather than single-adult, households.

Four experiments were conducted in San Diego county, California, in two counties in Alabama, and in Washington.\(^9\) The data set we use is from the cash-out experiment in San Diego county. For the experiment, 600 families were selected at random from the food stamp-receiving population and their benefits were converted from coupons to cash, sent in the form of a check. An additional 600 families, who continued to receive benefits in the form of coupons, were selected as a control and comparison group. The families were interviewed twice several months after the cash-out was implemented. Unlike other studies of food stamp participant behavior, the food stamp benefit data is taken from program records and matched with survey participants. For the purpose of what follows, we will refer to food purchased at a store for preparation and eating at home as food expenditure.\(^{10}\) This one-time survey of participants does not allow us to follow families who have switched from stamps to checks.

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\(^8\) For a criticism of Levedahl’s formulation, see Breunig and Dasgupta (2002). Ohls et al. (1992, p.100), using the same data set as ours, show that there is no difference in the number of trips to the store and the timing of trips to the store between check and stamp-receiving households in this data.

\(^9\) The experiments are described in Fraker, Martini, and Ohls (1995). These were the first large-scale experiments replacing food stamps with cash to be conducted in the United States.

\(^{10}\) This somewhat restricted definition matches the purpose of the food stamp program, which is to provide income for families to purchase groceries that they will use for meal preparation at home. It is generally not possible to use food stamps to purchase prepared or take-away food.
However, since the participants in the program were selected at random, comparison across the group of households which received checks and that which received stamps may give some preliminary indication of the presence of the cash-out puzzle. Despite the similarities in the stamp household and check household samples in terms of total income and receipt of government benefits, there is a significant difference in mean weekly food expenditure between the two groups.\textsuperscript{11}

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Food Expenditure for Stamp and Check Households</th>
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<tr>
<td></td>
<td>Weekly food expenditure per member of the food consumption unit</td>
</tr>
<tr>
<td>Stamp Households</td>
<td>$21.38</td>
</tr>
<tr>
<td>Check Households</td>
<td>$20.23</td>
</tr>
<tr>
<td>Difference</td>
<td>-1.15*</td>
</tr>
</tbody>
</table>

Normalizing expenditure by adult male equivalent\textsuperscript{12} for calorie intake or by the size of the food consumption unit provides essentially the same result. Only about 5\% of the households in this sample are constrained, thus we would not expect the measurable difference in food expenditure to be caused by the elimination of the constraint for the check-receiving households.

This comparison exploits the randomization in the experiment, but does not control for any characteristics that might differ between the two groups. The cash-out puzzle has more frequently been analyzed by estimating Engel curves for food expenditure for unconstrained stamp-receiving households. For our purposes, this strategy also seems natural because our primary interest lies in investigating the relative impact of in-kind welfare income vis-à-vis market income. Using two different specifications, we find that the marginal propensity to consume food out of stamps is significantly larger than that out of cash income. We use a linear-in-log (double log) model,

\begin{align*}
\ln(y_i) &= \alpha + \beta \ln(\theta_i + s_i) + \gamma \frac{s_i}{(\theta_i + b_i)} + X_i'\delta
\end{align*}

and the Working-Leser model, allowing different effects of cash income and food stamp benefit income

\textsuperscript{11} The data was gathered in two interviews, the second of which was a follow-up to ensure participants’ ability to recall the necessary information. Observations for which there was no follow-up interview or which indicated that the data was of dubious quality were dropped from the analysis, following what others have done. For regression estimation, we drop 16 observations (9 single-adult and 7 multiple-adult households) where food share of total expenditure is greater than one. Levadahl (1995) also dropped these observations. For all parameter estimates in the paper, "*" and "**" indicate significance at 90\% and 95\% level respectively. Numbers in parentheses below coefficient estimates are standard errors.

\textsuperscript{12} We use household size measured in equivalent nutrition units for food energy, an adult equivalent adjusted for guest meals and number of meals eaten at home. The means are similar to those reported by Fraker, Martini, and Ohls (1995) where it appears that they use this particular normalization.
(b) \[ w_i = \alpha_1 + \alpha_2 \ln(\theta_i + \alpha_3 s_i) + X_i' \delta \]

where \( y \) is food expenditure, \( w \) is share of food expenditure in total expenditure, \( \theta \) is cash income, \( s \) is stamp benefits and \( X \) is a vector of household characteristics.\(^{13}\)

We control for household size, receipt of other food gifts and subsidies, household composition and meals eaten outside of the household by household members and within the household by guests. Many of the control variables are not significant, but we leave them in the regression for comparability with other studies. A simple specification including only the household size and composition controls was estimated, but the main results are the same (see footnote 14). Full regression results are provided in the appendix in Tables A1 and A2. The primary results are summarized in Table 2.

<table>
<thead>
<tr>
<th>Table 2 Unconstrained stamp households</th>
<th>Linear-in-logs Model</th>
<th>Working-Leser Model</th>
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</thead>
<tbody>
<tr>
<td>MPC(s)</td>
<td>0.582**</td>
<td>0.489**</td>
</tr>
<tr>
<td></td>
<td>(0.108)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>MPC((\theta))</td>
<td>0.089**</td>
<td>0.141**</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>MPC(s) - MPC((\theta))</td>
<td>0.494**</td>
<td>0.348**</td>
</tr>
<tr>
<td></td>
<td>(0.112)</td>
<td>(0.044)</td>
</tr>
</tbody>
</table>

MPC(s): Marginal propensity to consume out of food stamp benefits
MPC(\(\theta\)): Marginal propensity to consume out of cash income
Sample size is 496. Standard errors are in parentheses.

Marginal propensity to consume food out of cash income is significantly lower than that out of market income, implying that a replacement of a dollar of stamp benefits by a dollar of market income will reduce household spending on food, even though the household is unconstrained.

Recall now the basic hypothesis we advanced in Section 1: households with multiple decision-makers would behave differently from households with one decision-maker. The household with one decision-maker should behave according to the standard model and treat cash income and stamps identically. If in fact intra-household dynamics are to explain the cash-out puzzle, it should be the case that the puzzle arises because of the behavior of households with multiple decision-makers. On the other hand, if factors other than intra-household dynamics are to explain the puzzle, then behavior of households with single decision-makers should also contribute to the puzzle to a significant extent.

\(^{13}\) The Working-Leser model is consistent with utility maximization and is the basis of the AIDS model. Chesher and Rees (1987) find that it fits food expenditure data better than linear or linear-in-logs specifications. Model (a) has been used in the food stamp literature by Senauer and Young (1986) and Levedahl (1995) and as Levedahl shows, this model imposes few restrictions on the relationships between the marginal propensities to consume out of stamps and income. We estimate (a) by OLS and (b) by maximum likelihood.
Table 3 provides a summary of the key regression results (full results are in the Appendix) when we estimate separate regressions for multi-adult and single-adult households. Pooling the two sub-groups and imposing identical response coefficients for the household characteristic variables leads to the same conclusion.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>San Diego Cash-out Experiment</th>
<th>Multiple-adult and single-adult headed households compared</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>All stamp households</td>
<td>Multi-adult households</td>
</tr>
<tr>
<td><strong>Linear-in-logs Model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPC(θ)</td>
<td>0.089**</td>
<td>.085**</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(.02)</td>
</tr>
<tr>
<td>MPC(s)</td>
<td>0.582**</td>
<td>.749**</td>
</tr>
<tr>
<td></td>
<td>(0.108)</td>
<td>(.137)</td>
</tr>
<tr>
<td>MPC(s) -</td>
<td>0.494**</td>
<td>.664**</td>
</tr>
<tr>
<td>MPC(θ)</td>
<td>(0.112)</td>
<td>(.142)</td>
</tr>
<tr>
<td><strong>Working-Leser Model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPC(θ)</td>
<td>0.14**</td>
<td>.148**</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(.02)</td>
</tr>
<tr>
<td>MPC(s)</td>
<td>0.483**</td>
<td>.505**</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(.035)</td>
</tr>
<tr>
<td>MPC(s) -</td>
<td>0.344**</td>
<td>.358**</td>
</tr>
<tr>
<td>MPC(θ)</td>
<td>(0.045)</td>
<td>(.038)</td>
</tr>
</tbody>
</table>

For single-adult households, marginal propensity to consume food out of food stamps is not significantly different from that out of cash income. For multiple-adult households, however, depending on the econometric model, the former is roughly 5 to 10 times the size of the latter. Hence, even if a dollar cutback in food stamp benefit is associated with, say, four additional dollars of market income, total food purchase by such households will still fall.14 The cashout puzzle seems to be

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14 We estimated simpler models using only income and household size—the substantive results are the same. We also considered alternative specifications of the Engel curve, both parametric and nonparametric. A simple bivariate nonparametric analysis casts serious doubt on a linear relationship between food expenditure and income. The linear model because is also objectionable on the grounds that it is inconsistent with a declining food share as income increases—a standard empirical regularity. We prefer the two specifications presented here, since both the \( \ln(\text{income})/\ln(\text{food expenditure}) \) and the \( \text{food share}/\ln(\text{income}) \) relationships appear roughly linear. (This nonparametric analysis is available from the authors.) We undertook extensive sensitivity analysis of our definition of "unconstrained" households and the classification of households into single and multiple decision
primarily driven by intrahousehold dynamics, rather than any other factor. We now proceed to provide a theoretical model which rationalizes this result.

3. Theory

Assume a household with two income earning members $M$ and $F$, children and (or) elderly dependent parents. Given any earning agent $k, k \in \{M, F\}$, we shall refer to the other earner as $-k$. Each agent $k$ consumes a composite private good $x_k$. The household receives a certain amount of another item in kind; either directly or, as in the case of food stamps, through vouchers that can be legally redeemed only through purchase of that item. Additional amounts of this item may however be purchased on the open market. Total household consumption of the item received in kind is $y$.

When the commodity transferred, by its very nature, can only be consumed by children or the elderly, or is consumed jointly by all household members (as with housing and many types of health inputs), it is intuitively straightforward to interpret it as a domestic public good. When it is alienable in character, as is, most importantly, the case with food, we shall assume that intra-household division of the commodity is determined according to some given sharing rule, whereby each agent’s allocation depends only on total household availability, $y$, and increases with it.\footnote{One may think of this in terms of a two-stage allocation process, whereby the household first collectively determines the quantities of private goods and the total quantity of food, and subsequently distributes the available food. This simplification allows us to address alienable and non-alienable goods in an identical manner. More complicated sharing rules, while compatible with our analysis, make the exposition cumbersome.} Thus, in this case, the commodity can be formally modeled as a domestic public good, though individual income earners need not (though they may) derive utility from other members’ consumption as well. While noting that the treatment is completely general, we shall henceforth refer, for convenience of exposition, to the item transferred as food, transferred through food stamps.

For notational simplicity, we normalize prices of all goods to unity. In the absence of the in-kind transfer program, total household market income would be $J$. We model the funding process used for the transfer program through the analytical convenience of a ‘welfare tax’ imposed on the household, the burden of which is distributed between $M$ and $F$ in some given way. Thus, when the household receives $s$ amount of food stamps, its market income is reduced by a more than equivalent amount, to $(J-s-d(s))$, where $d(s)$ is the deadweight loss imposed by the process of funding and running the welfare program, $d(0) = 0$. We assume that the marginal deadweight loss is positive but finite, i.e., $d'(s) \in (0, \infty)$. Total household income, from welfare and market sources combined, is $(J - d(s))$. A welfare cutback (equivalently, a market cash-out), is a reduction in stamp income, i.e., in the value of $s$. By reducing the associated deadweight loss, this necessarily increases total household income from welfare and market sources combined.
income.  The maximum amount of food stamps that can be provided is $\sigma$, which depends on $J$ according to $[\sigma + d(\sigma) = J]$, and $\sigma(J) \in (0, J)$. Food stamps cannot be resold for cash.  

A. Household decision-making:  

Let income earner $k$'s preferences be represented by a strictly quasi-concave utility function $U^k(x, y)$, $k \in \{M, F\}$. Each agent $k$ has (post welfare tax) cash (market) income $r^k$, $r^k \geq 0$; 

$$r^M + r^F = J - s - d(s). \quad (3.1)$$ 

In the absence of the food stamp program, taking the male-female earning ratio as given, agent $k$'s income would only depend on total household income, $J$. However, the welfare tax used to fund the commodity transfer may also reduce $k$'s market income. Hence, letting subscripts denote the corresponding derivatives, we write: 

for all $k \in \{M, F\}$:  

$$r^k = r^k(J,s), \quad r^k(J,s) > 0 \text{ if } s \in [0, \sigma(J)], \text{ and }$$  

$$r^k_{s}(J,s) \in [-1 - d'(s),0]. \quad (3.2)$$ 

It must be that $[r^k(J,\sigma(J)) = 0]$ and $[r^M_s(J,s) + r^F_s(J,s) = (-1 - d'(s))]$. 

Agent $k$ takes the other income earner’s contribution to household food purchase, $y_{-k}$, and the availability of food from food stamps, $s$, as given, and chooses the allocation of his/her own income between food, $y_k$, and the private good, $x_k$. Since $y$ is total spending on food, we have: 

$$y = y_{-k} + y_k + s.$$ 

Thus, agents play a Cournot game with respect to choice of contributions towards the domestic public good. We assume the existence of a Nash equilibrium. Agent $k$’s problem then is the following. 

16 We assume a positive marginal deadweight loss because, as discussed earlier in Section 1, we explicitly wish to examine the case for maintaining welfare programs even when their adverse general equilibrium effects reduce total earnings of poor households. The case of zero marginal deadweight loss is identical to the case where the state has a given amount, which it can transfer to the poor household either in cash or in kind. We discuss this case in Section 4A below.  

17 Commodity transfers are often provided at positive, but subsidized, prices. This is for example the usual case with food provided through public distribution systems in developing countries. Allowing positive prices, and modeling welfare cutbacks as reductions in the subsidy rate, complicate the algebra but do not change the substantive analysis. We discuss the consequences of allowing resale (say, in a black market) in Section 4b below.  

18 It is possible that, due to prevailing social norms, women may be obliged to transfer discretionary control over some, or most, of their market earnings to men. In that case, we identify $r^k$ with that amount over which agent $k$ maintains discretionary control, and assume that this amount increases with his/her market income.  

19 Whether agents shop for food separately or whether agents first pool contributions and, subsequently, one of them shops for the entire household is irrelevant for our analysis.  

20 Multiple public goods, while complicating the notation, do not add anything to the argument. Intra-household interaction may alternatively be modeled in Stackelberg fashion without affecting the conclusions. Udry’s (1996)
Max \( U^k(x_k, y) \),

subject to the budget constraint:

\[ x_k + y = r^k(J, s) + s + y_{-k}, \quad (3.3) \]

and the additional constraint:

\[ y \geq y_{-k} + s. \quad (3.4) \]

(3.4) combines two restrictions: (a) food stamps cannot be resold for cash, and (b) no agent can divert money allocated by the other agent for food purchase to his/her own private consumption.

Then, the solution to agent \( k \)'s optimization problem, subject to the budget constraint (3.3) alone, yields the optimal levels of \( y \) and \( x_k \) as functions of total income from all sources, i.e., of \([r^k(J, s) + s + y_{-k}]\). Let these unrestricted individual demand functions be given by:

(i) \( y = g^k\left( r^k(J, s) + s + y_{-k} \right) \), and (ii) \( x_k = h^k\left( r^k(J, s) + s + y_{-k} \right) \).

A3.1: For all \( k \in \{M, F\} \), \( g^k \) and \( h^k \) are continuous and increasing in \([r^k(J, s) + s + y_{-k}]\).

By A3.1, all goods are normal goods in the standard sense, which suffices to ensure the uniqueness of the Nash equilibrium. Then, the Nash equilibria yield single-valued household demand functions.

\[ x_k = x^k(J, s), \quad (3.5) \]

and

\[ y = y_k + y_{-k} + s = y(J, s). \quad (3.6) \]

It follows from (3.2), (3.3) and (3.4) that, in any Nash equilibrium, for all \( k \in \{M, F\} \):

\[ y(J, s) = \max\left\{ g^k\left( r^k(J, s) + s + y_{-k} \right) s + y_{-k} \right\}, \quad (3.7) \]

and

\[ x^k(J, s) = \min\left\{ h^k\left( r^k(J, s) + s + y_{-k} \right) r^k(J, s) \right\}. \quad (3.8) \]

Agent \( k \) is non-contributory in a Nash equilibrium if, in that Nash equilibrium, \( y_k = 0 \), and contributory otherwise. The household is constrained in a Nash equilibrium if, in that Nash equilibrium, both agents are non-contributory, i.e., if \( y(J, s) = s \), and unconstrained otherwise.

For all \( k \in \{M, F\} \), and for all \( J > 0 \), let \( V^k(J) \) be defined as the solution to:

\[ \left[ g^k\left( r^k(J, V^k) + V^k \right) = V^k \right]. \quad (3.9) \]

---

21 See Bergstrom et al. (1986).
Suppose that agent -k spent his entire income on his own private good. Then, if the household received \( s \) amount of food stamps, the optimal amount of food expenditure, from \( k \)'s point of view, would be \( g^k(J, s)+s \). \( V^k \) is simply that value of \( s \) for which this optimal amount is exactly equal to the amount of stamps actually provided. We first note that \( V^k \) must exist and be unique.

**Lemma 3.1.** Given \( A3.1 \), for all \( J>0 \), and for all \( k \in \{M, F\} \), \( V^k(J) \) is well defined; furthermore, \( V^k(J) \in (0, \sigma(J)) \).

**Proof:** See the Appendix.

B. Cost sharing:

We are now ready to introduce our key assumption.

**A3.2.** Given any \( J>0 \): (i) \([V^M(J) \neq V^F(J)]\); and (ii) if, for some \( k \in \{M, F\} \), \([V^k(J) > V^{-k}(J)]\), then \([r^k(J, s)+s]\) is increasing in \( s \) in the interval \([0, V^k(J)]\).

The restriction \([V^M \neq V^F]\) imposes heterogeneity in preferences and/or access to income between market participants. To see how weak this restriction is, note first that \([V^M \neq V^F]\) even if agents have identical preference orderings, so long as total household market income is distributed unequally. Conversely, even if household market income is distributed equally, this condition can be satisfied when agents have different preference orderings. Of course, it can also be generated by differences in both preferences and access to cash, combined in various ways.

Now, starting from an initial amount of food stamp income \( \tilde{s}, (\tilde{s} = \max\{V^M, V^F\}) \), consider a decrease in provision of food stamps by one dollar. This increases household market income by one dollar plus the marginal deadweight loss, i.e., by the amount \((1 + d')\). Unless the marginal deadweight loss is extremely high, i.e., unless \([d' \geq 1]\), at least one agent must necessarily gain less than one dollar of market income from each dollar of reduction in food stamps. Hence, since \( r^k_s \) is continuous in \( s \), there must exist some value of food stamp income, say \( s^* \), such that, for at least one agent, \([r^k(s)+s]\) is increasing in \( s \) in the interval \([s^*, \tilde{s}]\). Part (ii) of A3.2 involves (a) identifying this agent as the one with the higher value of \( V \), and (b) assuming \([s^* = 0]\). While (b) is essentially made for convenience, the key component of A3.2(ii) is in fact (a).

To see the intuitive justification for (a), first consider the case where both men and women bear significant portions of the marginal cost of the welfare program. When the marginal deadweight loss is moderate, i.e., when \([d' < 1]\), and the marginal cost of welfare payments, \([1 + d']\), is shared equally, \([r^k(s)+s]\) must be necessarily be increasing in \( s \) for both agents. This will hold even with unequal cost incidence, so long as each agent’s share of the marginal cost, \( r^k_s \), is more than the marginal
deadweight loss. The lower the marginal deadweight loss, the higher the extent of inter-gender inequality in cost sharing that one can allow. Thus, when either the marginal deadweight loss is relatively low or inter-gender cost incidence is relatively equal (or both), it is likely that, in every beneficiary household, each income earning individual will lose less than one dollar for every additional dollar of food stamp payment. Clearly, this in turn implies A3.2(ii), a weaker requirement.

Note now that, if, instead, the cost of the welfare program is borne overwhelmingly by one particular gender, say men, then \( r^k(s) + s \) will be increasing in \( s \) for women. A3.2(ii) will then be satisfied for households where \( V^F > V^M \). A priori, there does not seem to be any reason why such households should constitute a negligible proportion of welfare recipients. Thus, in either case, it seems quite plausible that A3.2(ii) will be satisfied in practice.22

C. Demand behavior:

We now proceed to address our central concern in three steps.

**Lemma 3.2.** Suppose A3.1 holds. Given any \( J > 0 \), let \( \bar{s}(J) = \max\{V^M(J),V^F(J)\} \). Then the household is: (i) unconstrained if \( s \in [0,\bar{s}(J)] \), and (ii) constrained otherwise.

**Proof:** See the Appendix.

Suppose the household is initially unconstrained. Then, by Lemma 3.2, it will remain so after a reduction in stamp income. Whether demand for food will fall remains as yet an open question.

**Lemma 3.3.** Suppose A3.1 holds. Given any \( J > 0 \), let \( \bar{s}(J) = \max\{V^M(J),V^F(J)\} \). Then, for any \( s^* \in [0,\bar{s}(J)] \), \( y(J,s) \) is increasing in \( s \) in the interval \( [s^*,\bar{s}(J)] \) if, and only if, for some \( k \in \{M,F\} \):

\[
[r^k(s) + s] \text{ is increasing in } s \text{ in the interval } [s^*,\bar{s}(J)], \text{ and } -k \text{ is non-contributory at } s^*.
\]

**Proof:** See the Appendix.

Consider an interval of food stamp values \( [s^*,\bar{s}] \). Suppose that, initially, the amount of stamps received is \( s^* \). Suppose further that one agent is non-contributory at \( s^* \). Then, by Lemma 3.2(i), the other agent must be contributory. Now consider an increase in household stamp income. The non-contributory member will remain so. The contributory member will reduce his/her cash contribution towards food purchase in response to the increase. Assume however that the conversion effectively increases the total income (cash and coupons) available to this agent. This will cause the contributory agent to reduce his/her contribution by less than the magnitude of the increase in food stamps. Hence, food purchase rises. The exact opposite happens when coupon income is reduced.

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22 Note that extremely high marginal deadweight loss, i.e. \( d' > 1 \), is necessary, but not sufficient, for \( r^k(s) + s \) to be decreasing in \( s \) for both agents.
Now suppose both agents contribute a positive amount initially. Then, if there were no deadweight loss, each agent would simply reduce his/her contribution by exactly that amount which he/she personally lost due to the increase in food stamps. This would keep household consumption of all commodities invariant. The additional, negative, income effect of the deadweight loss will however reduce the consumption of all commodities, as predicted by the traditional model.

What guarantees an interval where one agent is contributory, but not the other? This is simply \( F^M \neq V^F \). The larger the difference between \( V^M \) and \( V^F \), the larger this interval. When \( r^k(s) + s \) is increasing in \( s \) for the contributory agent, this condition is necessary as well.

**Lemma 3.4.** Suppose A3.1 holds. Given any \( J>0 \), let \( \bar{s}(J) = \max \{ V^M(J), V^F(J) \} \). 

(i) Suppose there exists \( s^* \in [0, \bar{s}(J)] \) such that, for some \( k \in \{ M, F \} \), \( [r^k(s) + s] \) is increasing in \( s \) in the interval \( [s^*, \bar{s}(J)] \), and \( -k \)-is non-contributory at \( s^* \). Then, for that \( k \), \( V^k(J) > V^{-k}(J) \).

(ii) If, for some \( k \in \{ M, F \} \), \( [V^k(J) > V^{-k}(J)] \) then, for that \( k \), \( -k \)-is non-contributory at all \( s \in [V^{-k}(J), V^k(J)] \).

**Proof:** See the Appendix.

Lemma 3.2(i) and Lemma 3.4(ii) together imply that, given A3.1 and A3.2(i), one agent must be non-contributory, and the other contributory, in the interval \( [\min \{ V^M, V^F \}, \max \{ V^M, V^F \}] \). We can now combine the results presented above to formulate our basic conclusion.

**Proposition 3.5.** Given any \( J>0 \), suppose, from some initial value \( s^* \), food stamp income of the household is reduced to \( s' : s' \in [\min \{ V^M(J), V^F(J) \}, \sigma(J)] \). Then, given A3.1 and A3.2, household consumption of food will fall.

Figure 1 relates household income and food expenditure with changes in (in-kind) welfare income. The topmost broken line represents total household income, while the unbroken line below represents food expenditure. The household is unconstrained in the region \( [\min \{ V^M, V^F \}, \bar{s}] \), while it is constrained in the region \( [\bar{s}, \sigma] \). Household marginal propensity to consume food out of market income is less than that out of welfare income in the entire interval \( [\min \{ V^M, V^F \}, \sigma] \).

Suppose that the reduced value of welfare income, \( s' \), lies within the interval \( [\min \{ V^F, V^M \}, \sigma] \). Then the welfare cutback will, by reducing household consumption of the domestic public good, thereby reduce the welfare of children and elderly dependants. If the proportion of households in this category is large, then a welfare cutback, even when generative of a large increase in market income, may have, in the aggregate, a significant negative effect on the well being of this, most vulnerable, section of the population.
In Figure 1, food consumption is increasing in $s$ throughout. In addition to A3.1 and A3.2, this requires that the agent with the lower value of $V$ be non-contributory even with zero welfare transfer. In general, it is possible (but not necessary) that both agents will turn contributory at some value below $\min\{V^M, V^F\}$. Then, further cutbacks in welfare provision will increase food expenditure. It is however easy to think of situations where, starting from an initial situation $s^* \geq \min\{V^M, V^F\}$, any cutback will generate an overall decline in food consumption.

If initially $V^k > V^{-k}$, then the interval $[V^{-k}, V^k]$ will widen with a relative increase in $k$’s income. This suggests the following. At any initial amount of welfare income $s^*$, most women will be non-contributory when the labor market is highly biased in favor of men. Consequently, the proportion of unconstrained households with non-contributory agents will be high. This proportion will initially fall with a reduction in the gender-based income differential, as more and more women turn contributory. Beyond a point, however, reductions in income differentials may increase the proportion of unconstrained households with non-contributory members, as an increasing number of men turn non-contributory. Hence, if $[r^k + s]$ is increasing in $s$ for both genders, the proportion of children and the elderly who will suffer due to welfare cutbacks would be higher when male-female income differentials are either high or relatively low, as compared to intermediate situations.

The problem arises because market gains ‘leak out’ to non-contributory agents. If, somehow, these could be restricted to the contributory agent, then A3.2(ii) would be violated, and the benefits of growth in household income will trickle down to children and the elderly. Empirical studies often find a relative increase in women’s market earnings to be associated with an increase in total household expenditure on children (and other domestic public goods). This suggests a case for male to female redistribution through interventions in the labor market as a means of reducing the adverse effect of welfare cutbacks on the well being of children. However, with prior distortions in the labor market, such measures can be counterproductive (Dasgupta (2000)). Even if one abstracts from this complication, three qualifications are still in order. First, these measures may generate significant deadweight losses. Second, they will reduce the welfare of children in households where women are non-contributory. Third, in households where both agents are contributory, they may reduce male contribution to such an extent that both women and children will be worse off (Dasgupta (2001)).

It seems reasonable to expect single-earner families to behave according to the unitary model, a view supported by our empirical analysis in Section 2. A welfare cutback may be expected to benefit

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23 There is evidence from developing countries that growth associated with a relative increase in women’s independent income may lead to male withdrawal from the responsibility of providing for household expenses. See Kabeer (1991) and da Corta and Venkateshwarlu (1997). The prevalence, in rich countries, of divorced/separated men who are non-contributory vis-à-vis their children may also be analyzed in this manner.

dependent members of such households if (a) the household is initially unconstrained, and (b) the cutback generates an overall increase in household income. However, such households may have lower market income. This is, for example, usually the case for single mothers. Consequently, a larger proportion of such households may in fact be constrained.\textsuperscript{25} Secondly, since a significant part of the cost of transfers to one-income families with dependants is actually borne by individuals without dependants, welfare cuts may actually reduce total income of the former. In either case, cutbacks will reduce the well being of children and the elderly even within single earner households.

D. Inter-gender redistribution:

Each dollar of welfare cut can be thought of as involving, for the contributory agent \( k \), (a) conversion of \((- r^k_s)\) dollars worth of welfare income to his/her own market income, and (b) loss of welfare income worth \([1 + r^k_s]\) dollars. Since, by A3.2(ii), \( [1 + r^k_s] > 0 \), and since, for a contributory agent, cash and commodity incomes are equivalent, cutbacks must reduce the welfare of this agent. However, cutbacks must also improve the welfare of the non-contributory agent. To see this, first note that (a) above, by itself, would keep household food expenditure, i.e., \([s + y^k_s]\), invariant. Hence (a) would keep the non-contributory agent’s welfare invariant as well. Now, since (b) reduces \( k \)’s private spending, it will reduce household food purchase by less than \([1 + r^k_s]\). Agent -\( k \) however gains, as additional market income, more than \([1 + r^k_s]\). Hence, his/her welfare must rise.

As noted earlier, in economies characterized by high male-female wage differentials and low female labor participation rates, women are more likely to be non-contributory. In these economies, in-kind transfers would seem to largely benefit men. Conversely, welfare cuts that improve women’s access to market income will improve their well being and reduce overall gender disparity. However, when earnings differentials are relatively low, and female labor participation rates high, in-kind transfers can be used to successfully ‘target’ welfare programs towards women.

Interestingly, when the proportions of men and women among non-contributory agents are relatively similar, welfare cutbacks may, on average, improve the well being of both men and women. A similar situation occurs when the male-female income differential is subject to cyclical fluctuations. For example, in many developing economies, men and women typically grow different crops,\textsuperscript{26} and are employed during different phases of the crop cycle. Hence, men may be non-contributory only during those parts of the year when their relative market earnings are low, and similarly for women. In these

\textsuperscript{25} In the food stamp data we use, over two-thirds of the constrained households are ones with single decision-makers, a larger proportion than in the unconstrained sample.

\textsuperscript{26} This is particularly true of sub-Saharan Africa. See dey Abbas (1997). Hopkins, Levin and Haddad (1994) using data from Niger, find that women tend to get small steady streams of income whereas men get seasonal work with large income followed by spells of no income.
contexts, if marketization of in-kind transfers improves lean period earnings of both genders, then, on average, all market participants may be better off, but at the cost of non-participants.

It has been noted that growth in household income is compatible with increasing male bias in resource allocation (and consequent female immiserization) inside the household (Kanbur and Haddad (1994)). The preceding discussion highlights instead the inherent ‘generation bias’ aspect of market activity, due to which even robust, pro-poor and gender-neutral economic growth may be associated with immiserization of generations which can not participate in economic activities.

4. Extensions

A. Cash versus commodity transfers:

The analysis remains unchanged when commodity payments are replaced by (exactly) equivalent cash welfare payments, rather than by additional market income. Formally, this extension to exact ‘welfare cash-outs’ simply involves assuming zero marginal deadweight loss in our model. In this case, a substitution of commodity payments by cash welfare payments must keep household consumption of the domestic public good invariant when both agents are contributory, unlike in our original formulation where this may rise. If both agents receive a positive share of any additional cash welfare income, then A3.2(ii) becomes redundant. The change in the form of welfare payments must reduce spending on the domestic public good in all households where at least one agent is non-contributory. This provides one possible explanation for the result noted in Table 1 of Section 2 above. If agents belonging to one particular gender (say, women) receive the entire cash payment, such spending will fall in households where women are non-contributory, while remaining constant in households where women are contributory and men non-contributory. Thus, with regard to the welfare of children and the elderly, in either case, a regime of commodity payments will Pareto-dominate one of equivalent cash welfare payments.

B. Cash welfare payments and resale:

With possible resale, commodity income can be converted to its equivalent cash welfare income. Hence, showing that a welfare cutback may reduce household spending on the domestic public good in this case amounts to establishing this possibility for cutbacks in cash welfare transfers.

27 Recent experiments in the US Food Stamps Program, substituting Electronic Benefit Transfers for traditional coupon payments, constitute an example. See Beecroft et al (1994) for details.

28 Except, in the second case, when all women are contributory. Standard justifications for implementing welfare transfers in kind, rather than in cash, are formulated by arguing that unlike cash, commodity transfers generate self-selection and thereby reduce the cost of screening potential beneficiaries (Blackorby and Donaldson (1988)). In our formulation, commodity transfers Pareto-dominate cash transfers because it is not possible for the state to screen out contributors from non-contributors.
The outcome depends on how control over cash welfare income is shared among income earners. Let agent $k$’s share of household welfare income $s$ be given by $\lambda^k(s)$, where,

for all $k \in \{M, F\}$, $[\lambda^k(0) = 0]$ and $\lambda^k \in [0,1]$.

First note that, with cash welfare payments, $[V^M \neq V^F]$ no longer suffices to ensure the existence of an interval where one agent is non-contributory, and the other contributory. To see this, suppose that $[V^M > V^F]$, and consider the polar case where the entire cash welfare payment is retained by $F$; i.e., where $[\lambda^F(s) = s]$. It follows from (3.9) that, in the Nash equilibrium at $s = V^F$, it may be the case that $[y_F > 0]$, (though, of course, $[y_F \leq s]$) and, hence, both agents may be contributory. We therefore need to replace part (i) of A3.2 by:

for some $k \in \{M, F\}$, $[g^{-k}(r^{-k}(s)) + g^{-k}(r^k(s))] \leq g^{-k}(r^k(s))$,

where $s = \min\{V^k, V^{-k}\}$. It is straightforward to show that (given A3.1) this condition implies, but is not implied by, $[V^k > V^{-k}]$.

Secondly, while, under commodity transfers without resale, the benefit accruing to either agent from an additional dollar of commodity income is exactly one dollar, under cash welfare payment it may be less. Furthermore, the household as a whole must always be unconstrained. We therefore need to replace part (ii) of A3.2 by the stronger assumption:

if, for some $k \in \{M, F\}$, $[V^k(J) > V^{-k}(J)]$, then $[g^{-k}(r^k(s)) + \lambda^k(s)]$ is increasing in $s$ in $[0, \sigma]$.

Given A3.1 and the stronger version of A3.2 specified above, our conclusions, as stated in Proposition 3.5, will hold even when transfers take the form of cash payments to the household.

5. Conclusion:

In this paper, we have examined a case for maintaining welfare and anti-poverty programs, even if their general equilibrium effects lead to net income losses for targeted households. Using a Cournot model of intra-household decision-making, we have shown that, under plausible restrictions on preferences and intra-household division of income, welfare cutbacks may reduce household spending on domestic public goods even while increasing total household income. This in turn would reduce the well being of children and elderly dependent members. We have found empirical support for our formulation in data generated by a cash-out experiment carried out in California. In the process, we have also provided an explanation for the so-called ‘cash-out’ puzzle, i.e., the higher marginal propensity to consume food out of food stamp income compared to that out of market income, frequently noted in empirical studies of the US Food Stamp program. Our results provide an argument against a ‘trickle down’, or growth oriented, view of intra-household distribution as it relates to inter-generation disparity, and point to a possible conflict between the interests of market participants on one hand and dependent non-participants on the other.
The empirical literature on intra-household distribution has typically concentrated on assessing the impact of changes in gender-specific income differentials on intra-household allocation. Our analysis points to the need for investigation of the extent of non-contribution, to domestic public goods, among income earners, and of the role played by welfare income vis-à-vis that by market income. The key issue here is whether the empirical regularities we have identified for the US Food Stamp case, which in turn underpin and motivate our theoretical formulation, can be generalized for other types of welfare programs, in developed as well as developing country contexts. Furthermore, we have restricted our attention to the pure income effect of a rise in returns to market participation, and thereby abstracted from possible substitution effects. Contexts under which substitution effects will strengthen/weak the income effect we have identified may be the subject of future research.

Appendix

Proof of Lemma 3.1.

Consider any \( J > 0 \) and any \( k \in \{ M, F \} \). First note that, since the private good is normal by A3.1, and (dropping \( J \) for notational simplicity), \( \left[ r^k(\sigma) = 0 \right] \), it must be the case that:

for \( s = \sigma \), \([g^k(r^k(s) + s) < s]\).

By assumption, \( r^k(0) > 0 \). By A3.1, therefore,

for \( s = 0 \), \([g^k(r^k(s) + s) > s]\).

It follows that \( V^k \), if it exists, must belong to the open interval \((0, \sigma)\).

Now, since \( g^k(s) + s \) is continuous in \( s \), noting that, by A3.1, \( g^k \) is continuous in its argument, we immediately have existence.

We now establish uniqueness. Suppose there exist solutions to (3.9), \( s^*, \hat{s} \in (0, \sigma) \) such that \([s^* > \hat{s}]\). Then, by (3.9) we have \([g^k(s^* + r^k(s^*)) > g^k(\hat{s} + r^k(\hat{s}))]\) which, by A3.1, implies:

\[ h^k(s^* + r^k(s^*)) > h^k(\hat{s} + r^k(\hat{s})). \]

Noting that \([h^k(\hat{s} + r^k(\hat{s})) = r^k(\hat{s})]\), and \([h^k(s^* + r^k(s^*)) = r^k(s^*))]\), we therefore have:

\[ r^k(s^*) > r^k(\hat{s}), \]

which is however possible only if \([s^* < \hat{s}]\). This contradiction establishes uniqueness. \( \diamond \)

Proof of Lemma 3.2.

Noting uniqueness of \( V^k \) by Lemma 3.1, by the same argument that establishes existence in Lemma 3.1, we also have:

\[ g^k(r^k(s) + s) > s \] for all \( s \in [0, V^k) \),

and
\[ g^k \left( r^k (s) + s \right) < s \] for all \( s \in (\bar{r}^k, \sigma) \). \hspace{1cm} (X2)

Putting \( \bar{s} = \max \{ \bar{r}^M, \bar{V}^F \} \), (X1) immediately implies part (i) of Lemma 3.2:

Now, by (X2), we have:

for all \( k \in \{ M, F \}, \left[ s > g^k \left( r^k (s) + s \right) \right] \) for all \( s \in (\bar{s}, \sigma) \).

Since the private good is a normal good by A3.1, it follows that:

for all \( k \in \{ M, F \}, \left[ y > g^k \left( r^k (s) + s + y_{-k} \right) \right] \) for all \( s \in (\bar{s}, \sigma) \). \hspace{1cm} (X3)

Noting that, by construction, \( [y(J, \bar{s}) = \bar{\bar{s}}] \), (X3) yields:

\[ [y(J, s) = s] \] for all \( s \in [\bar{s}, \sigma] \).

This establishes part (ii) of Lemma 3.2.

\[ \Box \]

\textbf{Proof of Lemma 3.3.}\n
Consider any \( s^* \in [0, \bar{s}] \). We shall first establish sufficiency. Suppose that, for some \( k \in \{ M, F \}, y^*_k = 0 \). Then, we have:

for some \( k \in \{ M, F \}, \left[ y^* \geq g^k \left( r^k (s^*) + y^* \right) \right] \). \hspace{1cm} (X4)

We shall first show that, given (X4), if \( [r^{-k} (s) + s] \) is increasing in \( s \), then the following must be true.

\[ [y = g^k \left( r^k (s) + y \right)] \] for all \( s \in (s^*, \bar{s}] \). \hspace{1cm} (X5)

Suppose not. Then, for some \( s \in (s^*, \bar{s}] \) \[ y = g^{-k} \left( r^{-k} (s^*) + s + y_{-k} \right) \]. \hspace{1cm} (X6)

Now, since \( [y^*_k = 0] \), by Lemma 3.2(i), we have \( [y^*_{-k} > 0] \), i.e.:

\[ [y^* = g^{-k} \left( r^{-k} (s^*) + s^* \right)] \]. \hspace{1cm} (X7)

by assumption, \( [r^{-k} (s) + s] \) is increasing in \( s \). Then, A3.1 and (X7) together imply:

\[ y > y^* \]. \hspace{1cm} (X8)

(X4), (X6) and (X8) together imply:

\[ [g^k \left( r^k (s) + s + y_{-k} \right) > g^k \left( r^k (s^*) + s^* + y_{-k}^* \right)] \]. \hspace{1cm} (X9)

(X9) and A3.1 together imply:

\[ [h^k \left( r^k (s) + s + y_{-k} \right) > h^k \left( r^k (s^*) + s^* + y_{-k}^* \right)] \]. \hspace{1cm} (X10)

Since, by assumption, we have:

\[ [h^k \left( r^k (s^*) + s^* + y_{-k}^* \right) \geq r^k (s^*)] \],

and, \( [r^k (s^*) \geq r^k (s)] \), it follows from (X10) that:

\[ r^k (s) < h^k \left( r^k (s) + s + y_{-k} \right) \],

which however contradicts (X6), thereby establishing (X5).
Since, by Lemma 3.2(i), for all \( s \in [s^*, \bar{s}] \), \( y > s \), it follows from (X4) and (X5) that,

\[
\text{for all } s \in [s^*, \bar{s}], \quad y = g^{-k} \left( r^{-k}(s) + s \right).
\]  

(11)

Since, by assumption, \( (r^{-k}(s) + s) \) is increasing in \( s \), (X13), and A3.1 together imply that \( y \) must be increasing in \( s \) in the interval \([s^*, \bar{s}]\).

We now establish necessity. We shall first establish that:

\[
\text{if } y \text{ is increasing in } s \text{ in the interval } [s^*, \bar{s}], \text{ then, for some } k \in \{M, F\}, \quad [y^*_k = 0].
\]  

(12)

Suppose not. Then, for all \( k \in \{M, F\}, \quad y^*_k > 0 \). We shall first show that:

\[
\text{there exists } \hat{s} \in (s^*, \bar{s}) \text{ such that, for all } s \in [s^*, \hat{s}], \text{ and for all } k \in \{M, F\}, \quad [y^*_k > 0]. \quad \text{(X13)}
\]

Since, for all \( k \in \{M, F\}, \quad y^*_k > 0 \), we have:

\[
g^k \left( r^k(s^*) + s^* + y^*_k \right) = [s^* + y^*_k + y^*_k^*].
\]

Then, by A3.1, it must be the case that:

\[
\left\lceil g^k \left( r^k(s^*) + s^* \right) - s^* \right\rceil < y^*_k.
\]

By construction,

\[
\left\lceil g^k \left( r^k(s^*) + s^* \right) - s^* \right\rceil < y^*_k.
\]

Then, noting that \( \left\lceil g^k \left( r^k(s) + s \right) - s \right\rceil \) is continuous in \( s \), it can be established, in a way exactly analogous to the way in which we established Lemma 3.1, that:

\[
\text{for every } k \in \{M, F\}, \text{ there exists a unique } t^k \in (s^*, \bar{s}) \text{ such that } [r^k(t^k) - h^k \left( r^k(t^k) + t^k \right) = y^*_k].
\]

Let \( \hat{s} = \min\{M, t^F\} \). Then, by an argument analogous to that used to establish (X1), we have:

\[
\text{for all } k \in \{M, F\}, \text{ and for all } s \in (s^*, \hat{s}), \quad [r^k(s) - h^k \left( r^k(s) + s \right) > y^*_k]. \quad \text{(X14)}
\]

Consider any \( s \in (s^*, \hat{s}) \). Suppose, for some \( k \), \( \left[ y^*_k = 0 \right] \). Then, it follows from (X14) that:

\[
y^*_k \geq y^*_k^* \quad \text{.} \quad \text{(X15)}
\]

Since, by assumption, \( y \) is increasing in \( s \) in the interval \([s^*, \bar{s}]\), we have:

\[
y > y^*. \quad \text{(X16)}
\]

By A3.1, (X16) implies:

\[
h^k \left( r^k(s) + s \right) > h^k \left( r^k(s^*) + s^* + y^*_k \right).
\]

Since \( \left[ r^k(s) \leq r^k(s^*) \right] \), this however implies the violation of (X15), thereby establishing (X13).

We now show that (X13) implies:

\[
y \text{ is decreasing in } s \text{ in the interval } [s^*, \hat{s}]. \quad \text{(X17)}
\]

Suppose that, for some \( s, s' \in [s^*, \hat{s}] \), \( s > s' \text{ and } y \geq y' \). Then, A3.1 and (X13) together imply:
for all $k \in \{M, F\}$, $[x_k \geq x_k']$. As $[J - d(s) < J - d(s')]$, we have a contradiction which establishes (X17). (X17) however violates our starting assumption. This establishes (X12).

Now, it can be established, in a way very similar to that used to establish (X5) that, if $y$ is increasing in $s$ in the interval $[s^*, \bar{s}]$, then:

if, for some $k \in \{M, F\}$, $[y_k^* = 0]$, then, for that $k$, $[y_k = 0]$ for all $s \in [s^*, \bar{s}]$. (X18)

Lemma 3.1(i), (X12) and (X18) together imply that:

if $y$ is increasing in $s$ in the interval $[s^*, \bar{s}]$, then for some $k \in \{M, F\}$,

(i) $[y_k^* > 0, y_{k-1}^* = 0]$ and (ii) $[y = g^k(r^k(s) + s)]$ for all $s \in [s^*, \bar{s}]$. (X19)

Given A3.1, it immediately follows from part (ii) of (X19) that $[r^k(s) + s]$ is increasing in $s$. This completes the necessity part of the proof, thereby establishing Lemma 3.3.

\[\diamondsuit\]

**Proof of Lemma 3.4.**

First suppose, given some arbitrary $J > 0$, for some $k \in \{M, F\}$, $[V^k \leq V^{-k} = \bar{s}]$. Suppose now that there exists some $s^* \in [0, \bar{s}]$ such that (i) $[y_k^* > 0, y_{k-1}^* = 0]$ and (ii) $[r^k(s) + s]$ is increasing in $s$ in the interval $[s^*, \bar{s}]$. Then, since $[V^* = s^* + y_k^*]$ and, by Lemma 3.2(ii) and Lemma 3.3, $[V^* < \bar{s}]$, we therefore have:

$[s^* + y_k^* < \bar{s}]$. (X20)

Using (X1) we then have: $[g^{k^*}((r^k(s) + y_k^*)y_{k-1}^* + (s^* + y_k^*)) > (s^* + y_k^*)]$. Since, by assumption, $[y_k^* > 0]$, using A3.1, we therefore get: $[g^{k^*}(r^k(s) + s^* + y_k^*) > s^* + y_k^*]$, which however implies $[y_{k-1}^* > 0]$. This contradiction implies $[V^k > V^{-k}]$, thereby establishing part (i) of Lemma 3.4.

Now suppose, for some $k \in \{M, F\}$, $[V^{-k} < V^k = \bar{s}]$. Consider any $s \in [V^{-k}, V^k)$. Using (X1), it must be the case that: $[g^{k^*}(r^k(s) + s) \leq s]$. Using A3.1, we then have:

$[g^{k^*}(r^k(s) + s + y_k^*) < s + y_k^*]$. (X21)

(X21) implies

$[y_{k-1}^* = 0]$. (X22)

Since, by Lemma 3.2(i), $[V > s]$, (X22) implies $[y_k^* > 0]$, which establishes part (ii) of Lemma 3.4. \[\diamondsuit\]

**Figure 1**

$y, J-d(s)$
References:


### Table A1
Comparing Multi-adult and Single-adult Unconstrained Stamp Households
Estimates for Linear-in-logs Model

Dependent variable is natural log of per-person food expenditure

<table>
<thead>
<tr>
<th></th>
<th>All Unconstrained Households n=496</th>
<th>Single-adult households n=281</th>
<th>Multi-adult households n=215</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(total income)</td>
<td>.472** (.056)</td>
<td>.253** (.097)</td>
<td>.544** (.069)</td>
</tr>
<tr>
<td>Proportion of Food Stamp Benefits in total income</td>
<td>1.511** (.343)</td>
<td>.452 (.537)</td>
<td>2.06** (.442)</td>
</tr>
<tr>
<td>ln(Household Size)</td>
<td>-.303** (.072)</td>
<td>-.211* (.124)</td>
<td>-.419* (.128)</td>
</tr>
<tr>
<td>Gift Food</td>
<td>.018 (.111)</td>
<td>-.003 (.014)</td>
<td>.067** (.018)</td>
</tr>
<tr>
<td>WIC Food</td>
<td>-.022 (.16)</td>
<td>-.021 (.19)</td>
<td>-.028 (.03)</td>
</tr>
<tr>
<td>School Breakfast subsidy (per child)</td>
<td>.003 (.028)</td>
<td>-.021 (.035)</td>
<td>.040 (.046)</td>
</tr>
<tr>
<td>School Lunch subsidy (per child)</td>
<td>.036** (.011)</td>
<td>.034** (.014)</td>
<td>.042** (.02)</td>
</tr>
<tr>
<td>Female-headed household (=1)</td>
<td>-.024 (.053)</td>
<td>-.082 (.121)</td>
<td>-.013 (.071)</td>
</tr>
<tr>
<td>Meals eaten as guest</td>
<td>-.043** (.008)</td>
<td>-.046** (.009)</td>
<td>-.029** (.014)</td>
</tr>
<tr>
<td>Meals eaten by guests</td>
<td>.036** (.006)</td>
<td>.036** (.008)</td>
<td>.037** (.009)</td>
</tr>
<tr>
<td>HH0_1 (Proportion of household members less than 1 year of age)</td>
<td>.15 (.185)</td>
<td>-.201 (.282)</td>
<td>-.224 (.332)</td>
</tr>
<tr>
<td>HH2_17</td>
<td>.169 (.152)</td>
<td>-.195 (.265)</td>
<td>.375 (.249)</td>
</tr>
<tr>
<td>HH61p</td>
<td>-.166 (.241)</td>
<td>-.157 (.345)</td>
<td>-.062 (.336)</td>
</tr>
<tr>
<td>constant</td>
<td>1.074** (.303)</td>
<td>2.343** (.52)</td>
<td>.737** (.393)</td>
</tr>
<tr>
<td>Adj. R²</td>
<td>.3883</td>
<td>.2899</td>
<td>.4662</td>
</tr>
<tr>
<td>MPC(s)</td>
<td>.582** (.108)</td>
<td>.213 (.172)</td>
<td>.740** (.137)</td>
</tr>
<tr>
<td>MPC(θ )</td>
<td>.089** (.016)</td>
<td>.064** (.029)</td>
<td>.085** (.02)</td>
</tr>
<tr>
<td>MPC(s)- MPC(θ )</td>
<td>.494** (.112)</td>
<td>.149 (.177)</td>
<td>.663** (.142)</td>
</tr>
</tbody>
</table>

*significant at 90% level
**significant at 95% level
<table>
<thead>
<tr>
<th></th>
<th>All Unconstrained Households n=496</th>
<th>Single-adult headed households n=281</th>
<th>Multiple-adult headed households n=215</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_2$</td>
<td>-.156** (0.017)</td>
<td>-.229** (0.032)</td>
<td>-.149** (0.018)</td>
</tr>
<tr>
<td>LN(Household Size)</td>
<td>-.074** (.022)</td>
<td>-.055 (.042)</td>
<td>-.11** (.04)</td>
</tr>
<tr>
<td>Gift Food</td>
<td>.007* (.004)</td>
<td>-.002 (.005)</td>
<td>.027** (.006)</td>
</tr>
<tr>
<td>WIC Food</td>
<td>-.009* (.006)</td>
<td>-.009 (.006)</td>
<td>-.013 (.01)</td>
</tr>
<tr>
<td>School Breakfast subsidy (amount per child)</td>
<td>-.003 (.01)</td>
<td>-.011 (.012)</td>
<td>.006 (.015)</td>
</tr>
<tr>
<td>School Lunch subsidy (amount per person)</td>
<td>.011** (.004)</td>
<td>.011** (.005)</td>
<td>.013* (.007)</td>
</tr>
<tr>
<td>Female-headed household (=1)</td>
<td>.003 (.017)</td>
<td>-.021 (.041)</td>
<td>.003 (.023)</td>
</tr>
<tr>
<td>Meals eaten as guest</td>
<td>-.012** (.003)</td>
<td>-.014** (.003)</td>
<td>-.006 (.005)</td>
</tr>
<tr>
<td>Meals eaten by guests</td>
<td>.013** (.002)</td>
<td>.013** (.003)</td>
<td>.013** (.003)</td>
</tr>
<tr>
<td>HH0_1</td>
<td>.022 (.058)</td>
<td>-.077 (.094)</td>
<td>.032 (.104)</td>
</tr>
<tr>
<td>HH2_17</td>
<td>.013 (.044)</td>
<td>-.088 (.088)</td>
<td>.073 (.079)</td>
</tr>
<tr>
<td>HH61p</td>
<td>-.085 (.031)</td>
<td>-.06 (.115)</td>
<td>-.073 (.111)</td>
</tr>
<tr>
<td>constant</td>
<td>1.008** (.086)</td>
<td>1.44** (.175)</td>
<td>0.975** (.097)</td>
</tr>
<tr>
<td>MPC(s)</td>
<td>.489** (.045)</td>
<td>.072 (.185)</td>
<td>.505** (.035)</td>
</tr>
<tr>
<td>MPC($\theta$)</td>
<td>.141** (.019)</td>
<td>.104** (.03)</td>
<td>.148** (.02)</td>
</tr>
<tr>
<td>MPC(s)-MPC($\theta$)</td>
<td>.348** (.044)</td>
<td>-.032 (.185)</td>
<td>.358** (.038)</td>
</tr>
</tbody>
</table>

See section 2 above for definitions of $\alpha_2$ and $\alpha_3$.

Estimates and standard errors for marginal propensities to consume are bootstrapped.

*significant at 90% level

**significant at 95% level