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Performance Evaluation for Long-term Investors

Dr Geoff Warren
Research Director
Centre for International Finance and Regulation

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Performance Evaluation for Long-term Investors

Dr Geoff Warren

Research Director
Centre for International Finance and Regulation
Geoff.Warren@cifr.edu.au

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Abstract

A performance evaluation approach is developed to support long-term investment programs, where asset selection is based on the internal rate of return estimated with reference to long-term cash flows. Returns are attributed into three components reflecting expected return, changes in discount rates, and changes in expected cash flows. Evaluating performance in this manner focuses attention on the key driver of long-term returns: cash flows generated over the long run, relative to the price paid.

Keywords: Long-term investing; performance attribution; portfolio construction

This article presents an approach to performance evaluation that is designed to support long-term investment programs based around discounted cash flow (DCF) analysis, to which long-term cash flow projections provide the key input. The approach is named “DCF based attribution analysis” (DBAA). It envisages an investment process under which assets are selected based on expected returns that are estimated as the internal rate of return (IRR) equating expected long-term cash flows with price. Subsequent performance is then evaluated by attributing realized returns into three components:

- (1) Expected return at the commencement of the period, i.e. initial IRR
- (2) Changes in expected returns, or discount rates
- (3) Changes in expectations for future cash flows

The aim is to establish the key driver of long-term returns – free cash flows generated over the long run, relative to the price paid for an asset – as the central focus of the investment process. Attribution of returns into the above three components facilitates such a focus through allowing emphasis to be placed on expected returns and cash flows, i.e. components 1 and 3. Meanwhile, the re-pricing effects related to changes in discount rates, as reflected in component 2, can be de-emphasized. Evaluating performance in this way sends a message to portfolio managers that they will be rewarded for investing in assets that offer high expected returns based on cash flow projections that are subsequently proven to be reliable if not conservative; and that they are not being held accountable for price fluctuations that occur for other reasons. Implementing DBAA should foster a long-term approach by encouraging fund managers to remain primarily concerned with cash flow fundamentals. It should diminish the attention paid to shorter-term price fluctuations and hence near-term returns, focusing on which may contribute to investor short-termism.

Certain features of DBAA are worth highlighting. While a long-term ‘buy-and-hold’ perspective is assumed when investing initially, it does not presume ‘set-and-forget’. Portfolios may be rebalanced in response to variation in expected returns through time that occur as a consequence of changes in prices or underlying cash flow fundamentals. Investors would sell assets where long-term expected returns have fallen, and buy assets which offer the highest long-term expected return. Another feature of DBAA is that it potentially links to an investment process whereby analysis and decision-making might be unbundled into three separate functions: projecting long-term cash flows and hence estimating expected returns; setting of required return hurdles; and the selection of assets to form portfolios. Also, DBAA is specified here at an elementary level, in order to draw out the key features. The presentation deals with a single review period. Important yet complex issues are discussed in general terms, such as the impact of intra-period cash flows and trading; tax and other costs; performance attribution over multiple periods; and real versus nominal effects.

This article proceeds as follows. After providing initial background, DBAA is specified in formal terms. An illustrative example is then presented. The impact of cross-border investment and hence currency is subsequently considered; with detail provided in the Appendix. The closing section discusses various issues with the approach, including implementation and limitations.

Background

The tendency for investors to adopt short investment horizons has been a topic of much discussion. Many commentators identify the manner in which investment managers are evaluated as contributing to a shortening of horizons (for example, see Curran and Chapple 2010; World Economic Forum 2011, 2012; Kay 2012; Papaioannou et al. 2013; CFA 2013). Specifically, the fact that performance evaluation and remuneration practices are based around relative returns over shorter periods is seen as encouraging investment managers to focus on short-term performance. DBAA offers a way of disrupting this short-term performance focus, by providing a mechanism to shift attention towards the drivers of long-term return generation. The underlying tenet is that the intrinsic value of an investment reflects its ability to generate future cash flows; and that initial expected return can be estimated by relating these cash flows to the price paid. DBAA unbundles returns in a manner that highlights the contributions that relate to the generation of long-term returns, versus those that are

relate to price fluctuations that occur in the interim. The aim is to divert attention away from the latter. DBAA formalizes an idea that is suggested in a report by the World Economic Forum (2011). It accords with the intuitions expressed by Jeffrey (1977) around the importance of income (earnings and dividends) for investors over longer horizons.

DBAA draws on the concept that returns can be divided into cash flow and discount rate effects. This concept has featured in the asset pricing literature for some time, with Campbell and Shiller (1998) providing a seminal contribution. Two other notable examples include Campbell and Vuolteenaho (2004), who re-examine the capital asset pricing model by dividing beta into ‘cash flow beta’ and ‘discount rate beta’; and Hecht and Vuolteenaho (2006), who analyze stock returns by estimating the same three components that form the basis of DBAA. Cochrane (2011), in his Presidential Address to the American Finance Association, acknowledges the value of delineating returns into cash flow and discount rate components, arguing that fluctuations in discount rates may be able to explain much about asset returns. Hence explaining realized returns as a consequence of the combination of fluctuations in the numerator (i.e. expected cash flows) and the denominator (i.e. discount rates) in the DCF equation has ample precedent and is soundly based in the theory of finance.

DBAA is closely related to the work of Cochrane (2014), who develops a model of asset pricing¹ and portfolio construction where assets are represented as a claim over a stream of expected long-term cash flows. Cochrane argues that there is considerable benefit in separating out and focusing on payoffs (i.e. cash flows) alone, and abstracting from the complications of dynamic portfolio strategy where the investor anticipates time-variation in expected returns and may hedge against changes in the investment opportunity set (see Merton 1973).² His underlying argument is that dynamic optimization is computationally hard, poorly understood and rarely used. Focusing on payoffs is a convenient way to simplify the problem. Cochrane’s approach amounts to a long-term multi-period analysis. It takes a step away from the usual practice in the asset pricing literature of examining single-period returns. DBAA might be viewed as a practical application that extends Cochrane’s method from the realms of asset pricing and portfolio selection, into the area of performance evaluation. Under DBAA, expectations for the stream of future cash flows (i.e. payoffs) not only provide the fundamental basis for the estimation of expected returns, but also the primary benchmark against which the skill of portfolio managers and analysts is evaluated.

DBAA is an extension on ‘Brinson’ attribution analysis: see Brinson and Fahler (1985) and Brinson, Hood and Beebower (1986), in which they seminally decompose single-period returns into contributions from asset allocation, sector and security selection, and market timing. Attribution analysis is a well-established and widely-used technique for identifying the contributions to portfolio returns, often relative to some benchmark. DBAA entails an attribution of returns into three alternative components of initial expected return, changes in discount rates, and changes in expected cash flows. It is motivated by, and consistent with, the literature on cash flows versus discount rate effects referred to above. Nevertheless, DBAA can still accommodate estimation of contributions along sector or asset lines, through simply summing the three components along these dimensions. For instance, contributions within each asset class might be accounted for as the weighted sum of the three components, allowing for better understanding of the underlying sources of performance within each asset portfolio.

Under DBAA, the first component of expected return at the beginning of a period forms a reference point. The other components capture the two ways in which return realizations may differ from the expected return: changes in expected cash flows, and changes in discount rates. The presumption is that changes in cash flows are of greater interest than changes in discount rates for investors that are primarily concerned with long-term returns. Revisions to expected cash flows represent a fundamental change in intrinsic value, and hence the

¹ Cochrane assumes preferences over mean and variance (i.e. quadratic utility) in order to derive asset pricing relations in closed form.

² While dynamic strategies do not appear under Cochrane’s specification, he notes that they might be analyzed as a separate payoff stream.

returns that are achievable from an investment over the long term. If the entire future stream of cash flows changes by 10%, then intrinsic value changes by 10%; and this represents a permanent change in value that in turn impacts on the long-term return that is ultimately realized. Cash flow changes are intimately related to the notion of risk as a ‘permanent loss of wealth’. Indeed, a key risk faced by long-term investors is mis-estimating long-term cash flows and hence underlying intrinsic value in the first place.

By contrast, price fluctuations arising from changes in discount rates can be interpreted differently. While changes in discount rates will influence realized returns achieved during a particular period, from the perspective of a long-term investor this does not necessarily undermine the rationale for the investment. Price fluctuations that are purely attributable to changes in the discount rate can be viewed as a ‘re-ordering’ of the sequence in which the returns are earned. If the discount rate goes up, a capital loss is incurred initially, but higher returns are subsequently earned off the lower price base. The equivalent happens when a sovereign bond with known cash flows is held to maturity, but bond yields fluctuate along the way. Yield fluctuations will impact on the market price of the bond and hence period-by-period returns in the interim. Nevertheless, the total return earned to maturity remains the same. Only the pattern of returns over time has changed.³

Estimating Expected Returns and Asset Selection

While the main aim of this article is to present DBAA, any attribution analysis should be linked to an overarching investment process. It is not the aim of this article to propose a comprehensive method of asset selection and portfolio construction, which may depend on the particular fund objectives and could entail a range of signals and constraints. Regardless, DBAA envisages an investment process under which assets are primarily selected based on long-term expected returns that are estimated as an IRR, i.e. the discount rate that equates current price with expected future cash flows. The expected return (i.e. discount rate, or IRR) at the commencement of a review period also provides a reference point for the decomposition of realized returns under DBAA. Further, the presentation in this article assumes that asset selection is guided by comparing expected returns with a required return, to generate an expected excess return. Portfolio construction is further discussed in the closing section.

Equation (1) sets out the calculation of the expected return its general form. Equation (1) is effectively a standard DCF equation, solved for the discount rate at the base period 0 (DR_0) that equates the current asset price (P_0) and expected future cash flows formed at period 0. ‘Cash flow’ (CF)⁴ is the cash flow available to investors.⁵ DR_0 will be taken as the measure of the long-term expected return at period 0.⁶ Equation (1), as well as the other equations that follow below, are specified for an individual asset, with a view to aggregating to the portfolio level.

³ Of course, fluctuations in bond yields and prices are not necessarily totally irrelevant, as they may imply an opportunity cost (or benefit) in timing of purchases. This relates to future changes in the investment opportunity set and dynamic investing, which are ignored under DBAA.

⁴ The formula assumes that cash flows occur at the end of each period.

⁵ DCF analysis and related cash flows may be specified in a variety of ways (see Koller et al. 2010). The presentation here is based around net cash flow distributed to investors. In the case of equities, this would amount to dividends less any contributed equity. Other definitions are possible, including free cash available for distribution plus value generated through reinvestment. This could potentially involve the use of earnings as a proxy for maintainable cash flows subject to certain assumptions, coupled with estimates to capture the value of growth opportunities.

⁶ Two points about an IRR estimated in this manner. First, IRR implicitly assumes reinvestment at the discount rate. Second, no allowance is made for expected time-variation in discount rates, i.e. any term structure for expected returns. The main implication is that IRR may vary from expected effective returns to the extent that an investor anticipates reinvesting cash flows at a different rate to the IRR. This issue is of limited consequence under DBAA, which is being applied over a single period during which reinvestment of cash flows is likely to be a minor influence on realized returns. However, it may impact on the integrity of the long-term expected return estimates. Nevertheless, while IRRs have potential to distort the magnitude of expected effective return to the extent that the reinvestment rate varies from the discount rate, it should leave the ranking of assets by expected return unaffected.

$$P_{t=0} = \sum_{t=1}^{\infty} \frac{E_0[CF_t]}{(1+DR_0)^t} \quad (1)$$

Where:

P = Price

CF = Cash flow

DR = Discount rate = internal rate of return (IRR)

t = Time subscript, with 0 indicating the base period

$E[.]$ = Expectations operator

The difference between DR and the required return (RR) provides an estimate of the long-term expected excess return (XR), as per equation (2). The portfolio manager might form their portfolio with reference to the XR offered by various assets.

$$E_0[XR_0] = E_0[DR_0] - RR_0 \quad (2)$$

Where:

XR = Excess return

RR = Required return

The estimation of required returns is an entire topic by itself that will not be discussed in detail. Nevertheless, it is envisaged that RR might be set in accordance with considerations such as overall portfolio objectives; the associated risk attached to each asset given those objectives; and perhaps the opportunity set of assets and returns structures available in the market at the time. For instance, RR may take into account the prevailing level of interest rates in specifying a risk-free or baseline rate of return, along with risk premiums that in turn could vary over time. RR for individual assets might be set with reference to some asset pricing model.

DCF Based Attribution Analysis (DBAA) - Derivation

The DBAA measures are now formally derived. While the derivation is with respect to a single asset, it is straightforward to aggregate across a portfolio and sub-groups of assets by applying asset weightings. The illustrative example presented in the next section indicates how this may be done.

The starting point for the derivation is equation (3), which is an expression for the realized return on an asset during period 1. For convenience, cash flow during period 1 (CF_1) is folded back into the total value at the end of the period 1 (V_1).⁷ Tax and other costs are ignored for convenience, although could conceptually be included in cash flows.

$$R_1 = \frac{P_1 + CF_1}{P_0} - 1 = \frac{V_1}{P_0} - 1 \quad (3)$$

Where:

R = Return

V = Value of investment at the end of the period

$0, 1$ = Indicator for end of period 0, 1

To establish a link between the return over period 1 (R_1) and the DCF relation, equation (4) is specified as the equivalent of equation (1) at the end of period 1. Under equation (4), DR_1 is the IRR conditional on expected

⁷ Implicitly this assumes “dividend irrelevance” (see Miller and Modigliani 1961).

cash flows at the end of period 1, which in turn may have been revised since the end of period 0. Note that the exponent on the denominator in equation (4) is $t-1$, which has the effect of reducing the discount rate applied to each cash flow term, including applying a discount rate of one to CF_1 .⁸

$$V_1 = \sum_{t=1}^{\infty} \frac{E_1[CF_t]}{(1+DR_1)^{t-1}} \quad (4)$$

Equation (5) represents the expectation at period 0 for value at the end of period 1 (V_1), under the assumption that neither cash flow expectations nor the discount rate change. $E_0[V_1]$ also equals the price at the end of period 0 (P_0) multiplied by one plus the discount rate, i.e. $(1+DR_0)$. The key takeaway is that realized return will equal the discount rate in the absence of any changes in expected cash flows or the discount rate.⁹

$$E_0[V_1] = \sum_{t=1}^{\infty} \frac{E_0[CF_t]}{(1+DR_0)^{t-1}} = P_0(1 + DR_0) \quad (5)$$

The return during period 1 (R_1) is now restated into the three DBAA components. The first step is to recast equation (3) by incorporating two offsetting terms in the numerator for expected value at the end of the period 1:

$$R_1 = \frac{E_0[V_1] + V_1 - E_0[V_1]}{P_0} - 1 \quad (6)$$

Substituting equations (4) and (5) into equation (6) produces equation (7):


$$R_1 = \frac{P_0(1+DR_0) + \sum_{t=1}^{\infty} \frac{E_1[CF_t]}{(1+DR_1)^{t-1}} - \sum_{t=1}^{\infty} \frac{E_0[CF_t]}{(1+DR_0)^{t-1}}}{P_0} - 1 \quad (7)$$

Rearranging:

$$R_1 = \frac{P_0(1+DR_0)}{P_0} - 1 + \frac{\sum_{t=1}^{\infty} \frac{E_1[CF_t] - E_0[CF_t] + E_0[CF_t]}{(1+DR_1)^{t-1}} - \sum_{t=1}^{\infty} \frac{E_0[CF_t]}{(1+DR_0)^{t-1}}}{P_0} \quad (8)$$

Finally, collecting terms and defining $E_0[R_1] = DR_0$ as the expected return over period 1, leads to equation (9). This is the DBAA equation and the key result:

$$R_1 = E_0[R_1] + \frac{\sum_{t=1}^{\infty} \left(\frac{E_0[CF_t]}{(1+DR_1)^{t-1}} - \frac{E_0[CF_t]}{(1+DR_0)^{t-1}} \right)}{P_0} + \frac{\sum_{t=1}^{\infty} \frac{E_1[CF_t] - E_0[CF_t]}{(1+DR_1)^{t-1}}}{P_0} \quad (9)$$



Realized Return = *Expected Return (DR₀)* + *Effect of Change in Discount Rate* + *Effect of Change in Expected Cash Flows*

⁸ Although CF_1 is known at period 1, we nevertheless retain the expectations operator to simplify the presentation.

⁹ This also assumes that any cash flows are reinvested at the discount rate, DR_0

On the right side of equation (9), the first term is the ‘expected return’ over period 1, $E_0[R_1]$. This equals DR_0 ; the IRR applying at period 0. It is the return that would be forthcoming under conditions where neither the discount rate nor expected cash flows are revised. The second term captures the contribution from revisions to the discount rate or the IRR.¹⁰ It is estimated as the change in present value at the end of period 1 arising from the discount rate moving from DR_0 to DR_1 , based the cash flows as expected at the end of period 0. The third term captures the contribution from revisions to expected cash flows, incorporating the realized cash flow for period 1. It is estimated as the impact of cash flow revisions on present value at the end of period 1, evaluated using the prevailing discount rate at the time, DR_1 . The second and third terms are both scaled by the opening price, P_0 . The three components sum to realized returns by construction.

Illustrative Example

Table 1 provides an illustrative example. The asset universe comprises five assets: three ‘risky’ assets with uncertain cash flows, a government bond and cash. Various components of the analysis are numbered as items (1) through (15) for reference. Table 1 is divided into three parts. Panel A presents numbers at the end of period 0 for the following: notional portfolio weights (item 1); expected and required returns (items 2 to 4); and expected cash flows (items 5 to 7). Panel B presents the equivalent numbers for the end of period 1. Panel C reports the performance evaluation for period 1, including: realized returns (item 8); DBAA estimates (items 9 to 12); and selected output from the underlying calculations (items 13 to 15).

Item (2) is the discount rate (DR) or IRR that is estimated conditional on the cash flow forecasts (items 6 and 7) and the observed price (item 5). The forecasts for expected cash flows at period 0 (panel A) and period 1 (panel B) comprise explicit forecasts over 5 periods (Item 6), and a continuing value (CV , item 7) at the end of period 5. The latter is estimated for the three risky assets using a standard constant growth discount model. The assumptions are listed under item (7), and include using RR as the discount rate in estimating CV .¹¹ The government bond is assumed to mature at the end of period 5. With regard to cash, the cash flows reflect the expectations for the returns from investing in cash on a ‘roll-over basis’ over the forecast horizon (see Warren 2007). This means that DR for cash equals the average expected return from holding cash over the entire forecast horizon. Note that the reported DR_0 for cash of 3.0% varies from the 2.5% cash rate for period 1. This establishes a difference between the long-term expected return on cash and the realized single-period return, which will be treated under DBAA as a discount rate effect.

To facilitate asset selection, the estimates for DR_0 and DR_1 (item 2) are compared with the required return (RR , item 3) for each asset, generating long-term excess expected returns (XR , item 4). The portfolio manager is assumed to set their notional portfolio weights (item 1) with reference to XR .¹² At the end of period 0, the portfolio manager has decided on a weighting of 35% each in risky asset 1 and risky asset 2, but 0% in risky asset 3 which offers a negative XR . The bond and cash offer expected returns in line with required returns, and are allocated 20% and 10% respectively. The asset allocation at the end of period 0 provides the weights by which the DBAA estimates for period 1 are aggregated. By the end of period 1, changes in prices and expected cash flows result in risky asset 2 and bonds becoming less attractive, while risky asset 3 now offers attractive excess returns following a sell-off. The portfolio manager responds by rebalancing the portfolio at the end of period 1 to reflect the updated long-term return expectations, switching from risky asset 2 to risky asset 3 and transferring some funds from the bond to cash.

¹⁰ The change in discount rates might be further decomposed into changes in the risk-free rate or some other reference asset, and changes in risk premiums.

¹¹ Use of RR in estimating CV is not germane to the example, and is convenient as it avoids iterative (and potentially unstable) solutions, and. It may be justified as representing the assumption that prices will converge towards trend valuations over time, as reflected in steady state cash flows discounted at the cost of capital.

¹² It is envisaged that the estimated excess returns are used as a guide to portfolio construction only. Other considerations would need to be taken into account, such as the nature of the mandate, broader portfolio objectives (e.g. real return targets), the need for diversification, and the level of confidence in the excess return estimates.

Table 1: DCF Based Attribution Analysis (DBAA) and Portfolio Selection – An Illustrative Example

A) PERIOD 0	(1)	(2)	(3)	(4)=(2)-(3)	(5)	(6)					(7)		
	Weight for	DR_0	Required	Excess Return	Price	Expected Cash Flows per Period					Continuing Value at end-Year 5		
	Period 1	(IRR_0)	Return, RR	$XR_0 = DR_0 - RR$	P_0	<u>1(E)</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>CV</u>	LT Growth	RR
Risky Asset 1	35%	12.3%	10%	2.3%	250	10.0	11.0	12.0	13.0	14.0	371	6%	10%
Risky Asset 2	35%	9.4%	8%	1.4%	200	10.0	10.5	11.0	11.5	12.0	247	3%	8%
Risky Asset 3	0%	9.3%	12%	-2.7%	400	10.0	12.0	14.0	17.0	20.0	540	8%	12%
Government Bond	20%	4.0%	4.0%	0.0%	100	4.0	4.0	4.0	4.0	4.0	100		
Cash	10%	3.0%	3.0%	0.0%	100	2.5	2.5	3.0	3.5	3.5	100		
Portfolio Total (per \$1)	1.00	8.7%	7.4%	1.3%	1.000	0.042	0.044	0.047	0.050	0.052	1.252		
<i>Risky Assets (per \$1)</i>	<i>70%</i>	<i>10.8%</i>	<i>9.0%</i>	<i>1.8%</i>	<i>0.700</i>	<i>0.032</i>	<i>0.034</i>	<i>0.036</i>	<i>0.038</i>	<i>0.041</i>	<i>0.952</i>		

B) PERIOD 1	Weight for	DR_1	Required	Excess Return	Price	Expected Cash Flows per Period					Continuing Value at end-Year 5		
	Period 2	(IRR_1)	Return, RR	$XR_1 = DR_1 - RR$	P_1	<u>1(A)</u>	<u>2(E)</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>CV</u>	LT Growth	RR
	Risky Asset 1	35%	16.3%	10%	6.3%	220	9.0	10.0	11.0	12.0	13.0	345	6%
Risky Asset 2	0%	5.4%	8%	-2.6%	250	10.2	10.8	11.5	12.0	12.5	258	3%	8%
Risky Asset 3	35%	20.0%	12%	8.0%	300	10.0	12.0	14.0	17.0	20.0	540	8%	12%
Government Bond	10%	2.7%	4.0%	-1.3%	105	4.0	4.0	4.0	4.0	4.0	100		
Cash	20%	2.9%	3.0%	-0.1%	100	2.5	2.5	3.0	3.0	3.0	100		
Portfolio Total (per \$1)	1.00	13.5%	8.7%	4.8%	1.000	0.035	0.039	0.044	0.049	0.054	1.473		
<i>Risky Assets (per \$1)</i>	<i>70%</i>	<i>18.1%</i>	<i>11.0%</i>	<i>7.1%</i>	<i>0.700</i>	<i>0.026</i>	<i>0.030</i>	<i>0.034</i>	<i>0.039</i>	<i>0.044</i>	<i>1.178</i>		

C) ATTRIBUTION	(8)	(9)	(10)	(11)	(12)=(9)+(11)-(3)	(13)					(14)	(15)	
	Return for	Attribution of Return for Period 1			$E[R]+AE[CF]$	Change in Expected Cash Flow					Δ Value End-Period 1 Due To:		
	Period 1	$E[R]$	ΔDR	$\Delta E[CF]$	less RR	<u>1(A)</u>	<u>2(E)</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>CV</u>	$\Delta DR E_0(CF)$	$\Delta E[CF] DR_1$
Risky Asset 1	-8.4%	12.3%	-13.4%	-7.3%	-5.0%	-1.0	-1.0	-1.0	-1.0	-1.0	-26.5	-33.4	-18.3
Risky Asset 2	30.1%	9.4%	15.7%	5.1%	6.4%	0.2	0.3	0.5	0.5	0.5	10.3	31.3	10.1
Risky Asset 3	-22.5%	9.3%	-31.8%	0.0%	-2.7%	0.0	0.0	0.0	0.0	0.0	0.0	-127.4	0.0
Government Bond	9.0%	4.0%	5.0%	0.0%								5.0	
Cash	2.5%	3.0%	0.4%	0.0%								0.4	
Portfolio Total (per \$1)	9.6%	8.7%	1.8%	-0.8%	0.5%	-0.001	-0.001	-0.001	-0.001	-0.001	-0.019	0.018	-0.008
<i>Risky Assets (per \$1)</i>	<i>10.9%</i>	<i>10.8%</i>	<i>1.1%</i>	<i>-1.1%</i>	<i>0.7%</i>	<i>-0.002</i>	<i>-0.001</i>	<i>-0.001</i>	<i>-0.001</i>	<i>-0.001</i>	<i>-0.027</i>	<i>0.011</i>	<i>-0.011</i>

Panel C presents the performance evaluation for period 1. Realized returns for each asset and the portfolio are reported under item (8). The portfolio generated a total return of 9.6%, which is greater than the expected return of 8.7% (see item 2, panel A) and exceeds the required return of 7.4% (see item 3, panel A). The return on each asset is attributed into the three DBAA components of expected return (item 9), the effect of changes in the discount rate (item 10), and the effect of changes in expected cash flows (item 11). The ‘dollar-value’ effect of changes in discount rates is reported under item (14). Similarly, the changes in expected cash flows per period for each asset are reported under item (13); and the aggregate ‘dollar-value’ effect is summarized under item (15). The estimates for individual assets are summed to generate the contributions both at the total portfolio level, and for the risky asset segment of the portfolio (see bottom of panel C). Under this illustrative example, the contributions sum to the total portfolio return by construction. In practice, there is likely to be a residual that reflects the effects of intra-period cash flows and trading, and costs that are not accounted for, and so on.

DBAA reveals that returns were boosted by changes in discount rates, which contributed +1.8% to total portfolio return and +1.1% to the return in the risky asset segment of the portfolio. By contrast, changes in expected cash flows subtract -0.8% from total portfolio return, and -1.1% from the return on the risky asset segment. Large downward revisions to cash flows for risky asset 1 more than offset the modest upward revisions for risky asset 2.

Item (12) reflects the excess return that has been realized over the period, abstracting from the effect of changes in discount rates. It is estimated by summing the contributions from expected return and changes in expected cash flows, and then deducting the required return. This is a key measure of the generation of long-term excess returns. On this measure, the total portfolio generates a positive increment of 0.5% over the required return for the period. Although this is lower than both the realized excess return of 2.2% (= 9.6% - 7.4%) and the 1.3% excess return that was expected (item 4, panel A), it is nevertheless positive. Hence the portfolio manager may still be judged as having added value, albeit less than would be implied by traditional return-based performance evaluation which focuses on the magnitude of realized (excess) returns in isolation. Ideally, item (12) would be examined over multiple periods, after which there should be greater visibility on whether expected cash flow forecasts are actually achievable.

In conclusion, under this illustrative example, DBAA reveals that the portfolio return has been boosted by fortuitous price fluctuations arising from reductions in discount rates. The latter portends reduced expected returns going forward. Meanwhile, the risky assets that were selected – most notably risky asset 1 – suffered a substantial downward revision in expected cash flows and hence attenuation of the long-term return that may be eventually realized. To the extent that the aim is to reward managers of long-term investment funds for correctly forecasting long-term return potential rather than short-term price fluctuations, any bonus might be calibrated accordingly.

Foreign Assets and Currency

The analysis has so far assumed that all values are expressed in a single currency. In practice, most portfolios contain assets denominated in multiple currencies. The existence of currency exposure complicates both the estimation of expected returns, and the attribution of realized returns. This section summarizes a treatment for foreign assets under DBAA, which is more formally specified in the Appendix. The method builds on Ankrim and Hensel (1994), in that it uses investment in foreign assets on a fully-hedged basis as a reference point. The method entails the initial projection of cash flows and estimation of expected return in foreign currency, which is then translated into a local currency equivalent by adjusting for exchange rate effects. As forward exchange rates are determined by interest rate differentials under covered interest parity, the reference point effectively becomes the foreign currency expected return scaled by the interest rate differential. Table 2 is a breakdown of equation (A7) from the Appendix, which is the derived expression under DBAA with respect to returns on foreign assets. Under this expression, the subscripts ‘*L*’ refer to local currency and ‘*F*’ to foreign currency; ‘*f*’ refers to the forward exchange rate premium or discount; ‘*cs*’ to currency surprise (defined as the deviation of the realized exchange rates from forward rates – see Ankrim and Hensel 1994); and *h* is the percentage of a foreign asset that is hedged, or the ‘hedge ratio’.

Table 2: DBAA Components for Foreign Assets

Component (<i>Local Currency Effect</i>)	Formula (<i>refer Appendix for additional detail</i>)
1. Expected return	$\left(1 + f_{\frac{L}{F},1}\right) \left(1 + DR_{F,0}\right) - 1$
2. Change in Discount Rate <i>May be further decomposed into:</i> (a) Hedged component (b) Interaction with currency surprise	$\left(1 + cs_1\right) \left(1 + f_{\frac{L}{F},1}\right) \frac{\sum_{t=1}^{\infty} \left(\frac{E_0[CF_{F,t}]}{(1+DR_{F,1})^{t-1}} - \frac{E_0[CF_{F,t}]}{(1+DR_{F,0})^{t-1}} \right)}{P_{F,0}}$ $\left(1 + f_{\frac{L}{F},1}\right) \left(\frac{\sum_{t=1}^{\infty} \left(\frac{E_0[CF_{F,t}]}{(1+DR_{F,1})^{t-1}} - \frac{E_0[CF_{F,t}]}{(1+DR_{F,0})^{t-1}} \right)}{P_{F,0}} \right)$ $cs_1 \left(1 + f_{\frac{L}{F},1}\right) \left(\frac{\sum_{t=1}^{\infty} \left(\frac{E_0[CF_{F,t}]}{(1+DR_{F,1})^{t-1}} - \frac{E_0[CF_{F,t}]}{(1+DR_{F,0})^{t-1}} \right)}{P_{F,0}} \right)$
3. Change in Expected Cash Flow <i>May be further decomposed into:</i> (a) Hedged component (b) Interaction with currency surprise	$\left(1 + cs_1\right) \left(1 + f_{\frac{L}{F},1}\right) \frac{\sum_{t=1}^{\infty} \frac{E_1[CF_{F,t}] - E_0[CF_{F,t}]}{(1+DR_{F,1})^{t-1}}}{P_{F,0}}$ $\left(1 + f_{\frac{L}{F},1}\right) \left(\frac{\sum_{t=1}^{\infty} \frac{E_1[CF_{F,t}] - E_0[CF_{F,t}]}{(1+DR_{F,1})^{t-1}}}{P_{F,0}} \right)$ $cs_1 \left(1 + f_{\frac{L}{F},1}\right) \left(\frac{\sum_{t=1}^{\infty} \frac{E_1[CF_{F,t}] - E_0[CF_{F,t}]}{(1+DR_{F,1})^{t-1}}}{P_{F,0}} \right)$
4. Contribution from unhedged portion of expected asset value	$cs_1(1 - h) \left(1 + f_{\frac{L}{F},1}\right) \sum_{t=1}^{\infty} \frac{E_0[CF_{F,t}]}{(1 + DR_{F,0})^{t-1}}$

Table 2 details the four DBAA components with respect to foreign assets, expressed in local currency terms. The additional fourth component addresses the impact of any unhedged exposure to the asset. Component 1 is the expected return at period 0, $E_0[R_{L,0}]$. It reflects the product of one plus the expected return in foreign currency ($DR_{F,0}$; the IRR in foreign currency), scaled by the premium or discount that would be secured via hedging over period 1 ($1 + f_{L/F,1}$). This treatment implicitly assumes that the forward exchange rate premium or discount over the hedging period is an acceptable approximation for the annualized forward rate over the long term; and that forward rates are an unbiased predictor of expected exchange rates, i.e. $E_0[cs_1] = 0$. Components 2 and 3 respectively capture the effect of changes in discount rates and changes in expected cash flows. The effect of these changes is initially estimated in foreign currency terms, which is then multiplied by $(1 + cs_1)(1 + f_{L/F,1})$. The latter amounts to one plus the exchange rate change over period 1. Thus hedging does not completely remove the impact of unexpected currency fluctuations under DBAA. Currency surprise still impacts on realized returns, and hence enters DBAA, due to an interaction with unexpected changes in asset values. An investor can only hedge a

specific asset value; thus any deviations from the expected asset value due to changes foreign discount rates or expected cash flows are effectively unhedged. Component 4 is the return from currency surprise arising from the unhedged portion of the expected asset value for end of period 1. Component 4 equals zero under full hedging.

In instances where the portfolio manager is not responsible for managing currency exposure, it may be useful to further break down component 2 and component 3 with a view to isolating all the impacts from currency surprise. Components 2a and 3a are the effects of changes in discount rates and expected cash flows respectively, adjusted for the forward hedging premium or discount. The manager might be considered fully accountable for these two sub-components, to the extent that the reference point is investment on a fully hedged basis. Components 2b and 3b are the residual effects related to currency surprise, including its interaction with the forward premium or discount. In situations where the manager is not responsible for managing currency, they should not be made fully accountable for these effects, which partly arise from unexpected currency movements.

Discussion

This closing section discusses various aspects of DBAA, including implementation issues and limitations. One feature is that the framework envisaged under DBAA accommodates analysis and decision-making being separated into three functions:

1. **Cash flow projection** – This function that might be performed by analysts, with independent review by portfolio managers. Forecasting of long-term cash flows is an important task that sits at the foundation of the approach.
2. **Specifying required returns** – This function would be subject to oversight and agreement with those responsible for establishing objectives at the total plan level. It would not be left up to the portfolio manager, given that they would have an incentive to reduce the hurdle. Depending on the governance structure, sign-off on required return targets could come from the governing Board, the Investment Committee, the Chief Executive Officer or the Chief Investment Officer. Managers of asset class portfolios, be they internal or external, might be provided with overall required return targets.
3. **Asset selection and portfolio construction** – This task should fall to portfolio managers exclusively.

Under the above separation of functions, analysts and portfolio managers could be evaluated in different ways. Analysts would be rewarded based on the accuracy of their cash flow forecasts across the universe of assets to which they are assigned.¹³ Analysts might be required to generate cash flow projections for all assets of potential interest, with a view to assisting portfolio managers in identifying a subset to be included in the portfolio.

Meanwhile, portfolio managers would be held accountable for the assets they select; and the effectiveness by which they form portfolios to meet long-term objectives. While the latter may encapsulate a broad range of considerations, DBAA would provide a key input by helping to gauge the extent to which the portfolio is generating excess long-term returns. A key measure would be the difference between the sum of expected returns and changes in expected cash flows, less the required return, i.e. item (12) in Table 1. Recall that this is a measure of returns abstracting from the impact of discount rate changes. With the change in expected cash flows a key component of this measure, focusing on it should make portfolio managers concerned with the reliability of cash flow forecasts for assets they include in the portfolio. Thus they would be required to scrutinize any projections provided by analysts. Focusing on returns abstracting from discount rate changes delivers the message that portfolio managers will be rewarded for identifying investments that offer attractive long-term expected returns based on cash flow forecasts that are plausible, if not conservative.

For example, assume two assets (A and B) both offer an expected excess return of 3%. Say the cash flow forecasts are subsequently revised upwards by 2% for asset A, but downwards by 4% for asset B. DBAA would reveal a

¹³ The cost of missed opportunities arising from overly conservative cash flow forecasts should also be considered.

realized excess return abstracting from discount rate effects of 5% for asset A, and -1% for asset B. The portfolio manager would be rewarded for selecting asset A, and penalized for choosing asset B. This possibility should encourage concern with the integrity of the cash flow forecasts when selecting assets; and consideration of the trade-off between the estimated expected return and the level of confidence that the baseline cash flow forecasts will be achieved or even exceeded. A ‘margin for error’ should be valued and appreciated. In sum, DBAA promotes favor for assets that offer attractive long-term expected returns which are in turn based on conservative forecasts.

One implementation issue is the time interval over which performance evaluation is conducted. This might align with the cycle of cash flow forecast revision and portfolio review. A quarterly interval would seem a good compromise under a long-term approach where portfolios are reviewed occasionally but not continuously. Shorter or longer review intervals should be feasible, such as a month, a year, or upon a major reassessment of the portfolio. One complication with longer intervals is that accounting for the effect of cash flows and transactions becomes more problematic. The equations as specified assume end-period cash flows, meaning that the effect of intra-period cash flows and trading (as well as any other unobserved influences, such as taxes, costs and data errors) can appear as an unexplained difference between the sum of the attribution components and the reported portfolio return. Providing the unexplained difference is not substantial, it might not be of great concern. If DBAA estimates are seen as approximations that provide one input into performance evaluation, then high accuracy may not be required.

Another implementation issue is aggregating performance contributions across time. This issue takes on heightened importance when investing for the long term, where the outcome achieved over an extended period is what matters most. It is also desirable to evaluate the integrity of long-term cash flow forecasts over longer periods: judgment on cash flows forecasts should be reserved until sufficient time has passed to test their validity, as far as feasible. Aggregation of contributions across time is a tricky issue, which will not be addressed in detail here. Nevertheless, a few observations are offered. DBAA generates arithmetic contributions, as the three components are constructed to sum to the total return. As a consequence, the problem emerges of aggregating arithmetic contributions across time in a manner that reconciles with the geometric returns earned at the total portfolio level. Menchero (2004) addresses this problem in some depth, and discusses some methods.

A feature of DBAA and its implied investment process is that a buy-and-hold mentality is presumed, with limited *anticipation* of the dynamic investment opportunities that may arise from changes in expected returns (i.e. changes in discount rates). In other words, it does not anticipate any changes in the investment opportunity set in the sense raised by Merton (1973). Equivalently, it does not acknowledge the value-add that may arise from holding off on asset purchases or sales in anticipation of a better price, which a long-term investor might validly contemplate with a view to maximizing long-term outcomes. Nevertheless, DBAA does not assume set-and-forget. Indeed, the approach accommodates a *reactive* dynamic element, as it envisages portfolio rebalancing as long-term expected return estimates are revised over time.

DBAA has its limitations. A key one is reliance on cash flow forecasts as the basis for estimation of expected returns and the subsequent attribution. The inherent subjectivity of cash flow forecasts leaves the decision process exposed to behavioral effects, such as optimism or confirmation biases. It might open up opportunities to game the system. For instance, there could be a tendency to escalate cash flow projections to justify investments in the first instance. More importantly, after an investment is made, there would be an incentive to avoid reducing cash flow forecasts, as this would subtract from the performance measures of interest. A number of strategies could help mitigate these problems. Separation of responsibilities should assist, by assigning the task of cash flow projection to analysts, and the task of review and asset selection to portfolio managers. Independent checks on the plausibility of the cash flow forecasts would be highly desirable, if not essential. It would also be advisable to reserve judgment until sufficient time has elapsed to evaluate the validity of the cash flow forecasts. For example, in the case of a greenfields infrastructure investment, cash flow projections might not be evaluated until after the project is completed and has been in operation for sufficient time to form an informed opinion about cash flow generating capacity.

A further limitation of DBAA relates to the possibility that cash flows and discount rates could be related in fundamental ways. For instance, a simultaneous rise in expected cash flow growth and discount rates could coincide with rises in real economic growth or inflation. DBAA would evaluate this combination of changes as enhancing long-term returns. This interpretation would be appropriate in the case of increased prospects for real growth. However, it would be misleading in the case of a rise in inflation. This raises the point that DBAA as presented here fails to distinguish between real and nominal effects; and is implicitly designed for a relatively stable inflation environment. Under a substantial shift in inflation, the output would need to be interpreted with care. Recasting DBAA into real and nominal effects represents a potential future extension.

Any performance evaluation system will have issues: none are perfect. Nonetheless, arguably the most important consideration when designing a system is the behaviors that it encourages *ex ante*. The key advantage of DBAA is that it provides an incentive to adopt a long-term focus when investing, especially relative to more traditional approaches where outcomes are heavily influenced by short-term price fluctuations.

References

- Ankrim, Ernest M. and Hensel, Chris R. (1994), "Multicurrency Performance Attribution", *Financial Analysts Journal*, 50(2), 29-35
- Brinson, Gary P. and Fachler, Nimrod (1985), "Measuring Non-U.S. Portfolio Performance", *Journal of Portfolio Management*, 11(3), 73-76
- Brinson, Gary P., Hood, L. Randolph and Beebower, Gilbert L. (1986), "Determinants of Portfolio Performance", *Financial Analysts Journal*, 42(4), 39-44
- Campbell, John Y. and Vuolteenaho, Tuomo (2004), "Bad Beta, Good Beta", *American Economic Review*, 94(5), 1249-1275
- Campbell, John Y. and Shiller, Robert, J. (1988), "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors", *Review of Financial Studies*, 1(3), 195-228
- CFA (2013) "Long-Term Financing: Investor Perspectives in Europe", *CFA Institute*, September
- Cochrane, John H. (2011), "Presidential Address: Discount Rates", *Journal of Finance*, 66(4), 1047-1108
- Cochrane, John H. (2014), "A Mean-Variance Benchmark for Intertemporal Portfolio Theory", *Journal of Finance*, 69(1), 1-49
- Curran, Ruth and Chapple, Alice (2011), "Overcoming the Barriers to Long-Term Thinking in Financial Markets", *Forum for the Future and Friends Provident Foundation*, Pixham End, Dorking, Surrey, UK
- Hecht, Peter and Vuolteenaho, Tuomo (2006), "Explaining Returns with Cash-Flow Proxies", *Review of Financial Studies*, 19(1), 159-94
- Jeffrey, Robert H. (1977), "Internal Portfolio Growth: The Better Measure", *Journal of Portfolio Management*, 3(4), 10-15
- Kay (2012), "The Kay Review of UK Equity Markets and Long-Term Decision Making: Final Report", *The UK Crown*, July
- Koller, Tim, Goedhart, Marc and Wessels, David (2010), *Valuation: Measuring and Managing the Value of Companies*, 5th Edition, John Wiley & Sons, Inc.
- Menchero, Jose (2004), "Multiperiod Arithmetic Attribution", *Financial Analysts Journal*, 60 (4), 76-91
- Merton, Robert C. (1973), "An Intertemporal Capital Asset Pricing Model", *Econometrica*, 41(5), 867-887
- Miller, Merton H. and Modigliani, Franco (1961), "Dividend Policy, Growth, and the Valuation of Shares", *Journal of Business*, 34(4), 411-433
- Papaioannou, Michael G., Park, Joonkyu, Pihlman, Jukka and van der Horne, Han (2013), "Procyclical Behaviour of Institutional Investors During the Recent Financial Crisis: Causes, Impacts, Challenges", *IMF Working Papers*, WP/13/193, September
- World Economic Forum (2011), "The Future of Long-term Investing" (with Oliver Wyman)
- World Economic Forum (2012), "Measurement, Governance and Long-term Investing" (March)

Appendix – Analysis of Foreign Assets

This Appendix derives the DBAA expressions for expected return and contributions to realized returns with respect to foreign assets. The treatment builds on Ankrim and Hensel (1994) by adopting investment on a fully hedged basis as the point of reference. While these expressions have a similar general form to those under a single currency, introducing exchange rates complicates the situation somewhat. The derived expressions reflect a process whereby cash flows and expected returns are projected in foreign currency terms; and then translated back into local currency by adjusting for exchange rate effects. When conducting DBAA, the deviations of exchange rates from forward rates (or ‘currency surprise’) still appear in the equations even under full hedging, due to an interaction with changes in asset values that arise as a consequence of changes to discount rates or expected cash flows. In effect, the investor hedges a specific asset value; and any deviations from the expected value are effectively unhedged.

Equation (A1) is the counterpart of equation (2) expressed in foreign currency. In this instance, the discount rate ($DR_{F,0}$) is the expected return or IRR in foreign currency (‘F’). Equation (A2) converts the foreign currency price ($P_{F,0}$) into local currency (‘L’) price at time 0 ($P_{L,0}$) through multiplying by the exchange rate, L/F_0 . The latter can be interpreted as the units of local currency that is received per unit of foreign currency.

$$P_{F,0} = \sum_{t=1}^{\infty} \frac{E_0[CF_{F,t}]}{(1+DR_{F,0})^t} \quad (\text{A1})$$

$$P_{L,0} = L/F_0 * P_{F,0} \quad (\text{A2})$$

Equation (A3) defines the expected exchange rate for period t as a combination of the baseline exchange rate in period 0 (L/F_0); the forward exchange rate premium or discount between period 0 and period t ($f_{L/F,t}$); and the expected deviation from the forward rate or ‘currency surprise’, denoted cs_t . This decomposition will be used in the estimation of expected returns and the other DBAA components in local currency. Note that under covered interest parity, the forward exchange rate will be determined by interest rate differentials, captured in the term, $\frac{1+r_{L,t}}{1+r_{F,t}}$ (with r referring to interest rates). Only cs_t is unknown at time 0.

$$E_0[L/F_t] = L/F_0 \left(1 + f_{\frac{L}{F},t} \right) (1 + E_0[cs_t]) = L/F_0 \frac{1+r_{L,t}}{1+r_{F,t}} (1 + E_0[cs_t]) \quad (\text{A3})$$

As mentioned, it is assumed that investment in foreign assets on a fully hedged basis provides the point of reference. The derivation will allow for estimation of the impact of any unhedged as a separate component.¹⁴ It is further assumed that the investor follows the practice of hedging asset values over one period, and rolling over the hedging contract.¹⁵ Equation (A4) is an expression for the expected value of the asset at the end of period 1. It is the counterpart of equation (4) under the single (i.e. local) currency case. Equation (A4) is formed by combining equations (A1), (A2) and (A3), under the assumption that $E_0[cs_t]=0$ under full hedging.

$$E_0[V_{L,1}] = L/F_0 \left(1 + f_{\frac{L}{F},1} \right) \sum_{t=1}^{\infty} \frac{E_0[CF_{F,t}]}{(1+DR_{F,0})^{t-1}} = P_{F,0} * L/F_0 \left(1 + f_{\frac{L}{F},1} \right) (1 + DR_{F,0}) \quad (\text{A4})$$

¹⁴ It is anticipated that currency decisions might be made and addressed separately, perhaps in relation to a policy benchmark that differs from fully hedged. This need not affect the manner in which total portfolio returns are decomposed under DBAA. However, a separate system may be required to specifically evaluate the effect of currency decisions relative to the policy benchmark.

¹⁵ The alternative would be to assume that the investor hedges the stream of expected cash flows. While this would have the theoretical attraction of translating the entire stream of expected cash flows into local currency terms, it seems more appropriate to adopt an approach that aligns with industry practice.

Equation (A5) represents the expected return on the asset in local currency over period 1. It reflects the product of (one plus) the long-term expected return in foreign currency, and the forward premium or discount secured by hedging over period 1. Given that forward exchange rates are determined by interest rate differentials, this is equivalent to scaling the expected return in foreign currency by the difference in interest rates between the two countries. For instance, if the local interest rates are higher than foreign interest rates, then hedging would augment the expected return in local currency. Equation (A5) is offered as a workable proxy for the long-term expected return, which in turn might be compared with a local currency required return on the asset.

$$E_0[R_{L,1}] = \frac{V_{F,0} * L / F_0 \left(1 + f_{\frac{L}{F},1}\right) (1 + DR_{F,0})}{P_{F,0} * L / F_0} - 1 = \left(1 + f_{\frac{L}{F},1}\right) (1 + DR_{F,0}) - 1 \quad (A5)$$

Attribution of realized returns into various components is now addressed. To derive the DBAA equations, an expression for the value at the end of period 1 is required. This appears in the form of equation (A6). The first term on the right hand side (top line) is the unhedged value of the asset, incorporating Equation (A3) so that the exchange rate at period 1 is presented as a combination of the exchange rate at period 0, the forward premium or discount, and the realized currency surprise. The second term (bottom line) captures the gain or loss from hedging h percent of the expected asset value at end of period 1. Recall that the ' $t-1$ ' exponent in the denominator acts to time-shift the discount rate, so that the realized period 1 cash flow is multiplied by one, and so forth.

$$V_{L,1} = \frac{L}{F_0} \left(1 + f_{\frac{L}{F},1}\right) (1 + cs_1) \sum_{t=1}^{\infty} \frac{E_1[CF_{F,t}]}{(1 + DR_{F,1})^{t-1}} - h cs_1 \frac{L}{F_0} \left(1 + f_{\frac{L}{F},1}\right) \sum_{t=1}^{\infty} \frac{E_0[CF_{F,t}]}{(1 + DR_{F,0})^{t-1}} \quad (A6)$$

Equation (A7) is the attribution equation for a foreign asset. It is derived in a similar manner to its local currency counterpart of equation (8), after rearranging terms and converting the base local price in period 0 into a foreign currency using: $P_{F,t-1} = P_{L,t-1} * F / L_0^{-1}$ (see equation (2A)). Equation (A7) has a similar form as equation (8), but includes exchange rate parameters and one additional component to capture the impact of any unhedged exposures. The first term is the expected return (see equation (A5)). The second term (top line) reflects the effect of changes in discount rates; and the third term (middle line) captures the change in expected cash flows. The main point of difference to the local currency case is that the innovations in expected cash flow and discount rates are initially estimated in foreign currency. They are then translated into the local currency equivalent by adjusting for currency effects. In general, the currency effects reflect the product of one plus the currency surprise and one plus the forward premium or discount, which amounts to the exchange rate change (refer equation (A3)). This reflects the fact any deviations from expected value are unhedged, regardless of whether the investor hedges. The fourth term (bottom line) captures the effect of any unhedged exposure, with reference to the expected asset value at the end of period 1. If the asset is 100% hedged ($h=1$), this term is zero. If the asset is unhedged ($h=0\%$), it accounts for the full impact of cs_t on local returns. Equation (A7) is decomposed further in Table 2.

$$R_{L,1} = E_0[R_{L,1}] + (1 + cs_1) \left(1 + f_{\frac{L}{F},1}\right) \frac{\sum_{t=1}^{\infty} \left(\frac{E_0[CF_{F,t}]}{(1 + DR_{F,1})^{t-1}} - \frac{E_0[CF_{F,t}]}{(1 + DR_{F,0})^{t-1}}\right)}{P_{F,0}} +$$

$$(1 + cs_1) \left(1 + f_{\frac{L}{F},1}\right) \frac{\sum_{t=1}^{\infty} \frac{E_1[CF_{F,t}] - E_0[CF_{F,t}]}{(1 + DR_{F,1})^{t-1}}}{P_{F,0}} +$$

$$cs_1 (1 - h) \left(1 + f_{\frac{L}{F},1}\right) \sum_{t=1}^{\infty} \frac{E_0[CF_{F,t}]}{(1 + DR_{F,0})^{t-1}} \quad (A7)$$