Royal Commission
into Aged Care Quality and Safety

THE COST OF RESIDENTIAL AGED CARE

TECHNICAL SUPPLEMENTARY REPORT 2: COST FRONTIER ANALYSIS OF AUSTRALIAN RESIDENTIAL AGED CARE FACILITIES

RESEARCH PAPER 9

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The Royal Commission into Aged Care Quality and Safety was established by Letters Patent on 8 October 2018. Replacement Letters Patent were issued on 6 December 2018, and amended on 13 September 2019 and 25 June 2020.

The Honourable Tony Pagone QC and Ms Lynelle Briggs AO have been appointed as Royal Commissioners. They are required to provide a final report by 26 February 2021.

The Royal Commission releases consultation, research and background papers. This research paper has been prepared by the University of Queensland for the information of Commissioners and the public. The views expressed in this paper are not necessarily the views of the Commissioners.

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Technical Supplement Report 2: Cost frontier analysis of Australian residential aged care facilities

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Abbreviations

ABS	Australian Bureau of Statistics
AIC	Akaike information criterion
ALS77	Aigner, Lovell and Schmidt (1977)
BC92	Battese and Coelli (1992)
BC95	Battese and Coelli (1995)
BIC	Bayesian information criterion
CFG95	Caudill, Ford, and Gropper (1995)
CI	Confidence interval
CSW14	Chen, Schmidt, and Wang (2014)
G05	Greene (2005)
JLMS	Jondrow et al. (1982)
K90	Kumbhakar (1990)
KLH14	Kumbhakar, Lien, and Hardaker (2014)
KW05	Kumbhakar and Wang (2005)
OBD	Occupied bed days
OLS	Ordinary least squares
PL81	Pitt and Lee (1981)
Q1	Quality level 1
Q2	Quality level 2
Q3	Quality level 3
RACF	Residential aged care facility
RVU	Relative Value Unit
SFA	Stochastic frontier analysis
SS84	Schmidt and Sickles (1984)
TSR1	Technical Supplementary Report 1
TSR2	Technical Supplementary Report 2
WH10	Wang and Ho (2010)

1. Context and structure of this report

The University of Queensland was commissioned by the Royal Commission into Aged Care Quality and Safety (the Royal Commission) to conduct an analysis to understand the efficient cost associated with providing residential aged care at different levels of quality.

To help achieve this aim, a composite indicator of quality was developed for use in the efficiency analysis. A series of comprehensive cost function and frontier analyses were then conducted to identify the cost frontier that best approximated the residential aged care data available in Australia. This frontier was then used to estimate the efficient cost to provide services at different levels of quality.

The ***Cost of Residential Aged Care*** **report** has two **technical supplementary reports**:

- ***Technical Supplementary Report 1 (TSR1): Composite index for quality of care in Australian residential aged care facilities.*** This report describes the data on quality used in the analysis, exploratory analyses and the methods to construct the composite quality index. It has two appendices:
    - *TSR1 Appendix A: Literature review on quality in residential aged care facilities*
    - *TSR1 Appendix B: Latent class analysis to construct the quality index at the provider level*
- ***Technical Supplementary Report 2 (TSR2): Cost frontier analysis of Australian residential aged care facilities.*** The report outlines the step-by-step approach to identify the cost functions that best represent the cost structure of the residential aged care industry in Australia. The main results and the sensitivity analyses conducted to ensure the robustness are described. It has three appendices:
    - *TSR2 Appendix A: Data diagnostics: Identifying and analysing outliers in cost and output data*
    - *TSR2 Appendix B: Robustness check of estimated inefficiency: Semi-parametric least squares stochastic frontier analysis*
    - *TSR2 Appendix C: Additional results from the stochastic frontier analysis*

A very important point we wish to highlight to readers from the outset is that the results sometimes include the differences in efficiency estimated between for-profit, not-for-profit and government facilities. These differences should be interpreted with great caution as they may (and perhaps are likely to) reflect differences in quality achieved by the different ownership types which have not been able to be distinguished within the three quality levels by the composite quality index.

2. Theoretical underpinnings

A standard approach in productivity and efficiency theory has been followed, in which it was assumed that the technology of the decision-making unit (i.e. aged care facility), can be characterised by their technology set Ψ_{it}, defined as,

$$\Psi_{it}(\mathrm{z}) = \{(x, y) : x \text{ can produce } y \text{ with knowledge available to } i \text{ } at \text{ } t \text{ and conditions } z\}$$

where $x \in \mathbb{R}_+^p$ is a vector of inputs that the decision-making unit uses to produce a vector of outputs $y \in \mathbb{R}_+^q$ with knowledge available to i at time t and subject to a set of conditions $z \in \mathbb{R}_+^d$ faced by i at t and assuming it satisfies standard regularity conditions of production theory. For more details, see Sickles and Zelenyuk (2019).

In reality, each decision-making unit i may have a different and very complex technology (which might be not fully clear even to the decision-making unit), perhaps impossible to formulate, especially because of lack of data on all the inputs, all the outputs and all the conditions. It is, therefore, necessary to make various simplifying assumptions on the model that are usually done in the related research fields. The goal is not to represent the reality fully (which is practically impossible), rather represent it the best it can be, focusing on the most essential part within the time and resources available to the authors, often sacrificing on many details that are considered less important. In this respect, it is worth reminding a famous quote from one of the greatest statisticians, George Box, on this matter:[1]

> **"Since all models are wrong the scientist must be alert to what is importantly wrong. It is inappropriate to be concerned about mice when there are tigers abroad."**

For the estimation, the principle of parsimony has been followed. That is, the model was built to ensure a desired level of explanation with as few predictor variables, and as intuitive, as possible, which increases in complexity when it results in substantial improvements of representing the reality.

After carefully analysing the goals of the study, the available data and existing studies, and based on the team's expertise, it was concluded that the main tool for the analysis should be the cost function (estimated via different methods) and all subsequent analysis were based upon it.

Formally, the cost function $TC: \mathbb{R}_+^q \times \mathbb{R}_+^p \times \mathbb{R}_+^d \to \mathbb{R}_+$ is defined as,

$$TC(y, w|\Psi_{it}(\mathrm{z})) = \min_{\mathrm{x}}\{w'x : (x, y) \in \Psi_{it}(\mathrm{z})\}$$

where, $w \in \mathbb{R}_{++}^p$ is the vector of prices faced by the decision-making unit, i.e., it reflects the optimal (minimisation of costs) choices of a decision-making unit i in period t with respect to the outputs that need to be produced, processes and conditions the decision-making unit faces at that time.

It is worth noting that the cost function is a fundamental concept in economics, which is theoretically well defined in general and relevant for all types of activities in particular, whether the decision-making units are a profit maximiser or a not-for-profit, or religious or government owned. Under standard assumptions, the cost function is known to be a dual and complete characterisation of technology set Ψ_{it}, in the sense that it contains all information about the corresponding technology that is in Ψ_{it} as well as the optimisation behaviour with respect to the input prices observed by decision-making units (and not observed by us, the researchers) and conditions.

The cost function is convenient because it is a natural aggregator of the many (disaggregated) inputs used in the production process, for which the data is typically unobserved (as in this case) implying that the estimation of the technology set or its frontier is practically infeasible. It is well studied in econometric analysis and in frontier analysis in particular.

Therefore, the first stage was to estimate the average tendency for this important economic relationship as accurately as practically possible. This served as the stepping stone for the second stage of this study, the frontier analysis to estimate the efficiency relative to the best practice.

Given the amount of variation (and potential noise) in the data, the first (and fundamental) step was to model it through regression analysis (parametric and, for robustness, nonparametric). That is, to look at the average tendency in the sample (conditional on various variables that are found to be significant from economic and statistical perspectives). The average tendency was examined between the total cost and the output (non-

[1] Box, G. E. P. (1976), "Science and statistics" (PDF), Journal of the American Statistical Association, 71 (356): 791–799.

adjust and adjusted), conditional on many factors. Following this, a standard approach in efficiency and productivity analysis for the frontier analysis was followed, where the (in)efficiency term is typically one for all types of inputs (or costs) or outputs, modelled as an overall and equiproportional residual (in addition to noise).

At the second stage, the inefficiency was analysed with the information from the first step, using frontier methods (stochastic frontier analysis) and to help estimate the efficient cost of delivering residential aged care for a range of output quantities and care qualities observed historically. The results from the average tendency (from the regression analysis) and from the frontier analysis are presented to provide an understanding of the gap between what is occurring in the industry on average (which, in principle, should be attainable by any facility, ceteris paribus) and what is occurring at the frontier (observed best practice, possibly attainable only by the best).

The association between quality levels, inefficiency and different cost categories (e.g. direct care, hotelling, accommodation, administration, and others cost) was of particular interest to the stakeholders of this project. Higher quality facilities were believed to have a more efficient cost of care than lower quality facilities, due to a better workforce, working environment and clinical governance. For facilities of different quality levels, the different cost categories were thought to possibly have different relationships with the output (e.g. when output increases, Q1 facilities might spend more on hotelling services than Q3 facilities). As such, in the third stage of the frontier analysis, separate frontiers for each cost category were estimated, with the quality indices included in the frontier functions. The estimation process was similar to that of the total cost, starting with the (average) cost function and statistical tests to examine whether or not inefficiency associated with each cost category existed, followed by the frontier estimation and statistical tests to confirm the existence of inefficiencies. Finally, cost inefficiencies (in Australian dollars) and inefficiency scores were calculated from the final models.

3. Regression analysis of the total cost function

3.1 Brief overview of data

The sample available for total cost analysis includes 6,188 observations across a 5-year period from financial years 2014/15 to 2018/19. The main variables used to estimate the cost function were total cost and output.

The unit of measurement for the total cost is Australian dollars ($) recorded at different periods, i.e., nominal values, which are deflated using the Wage Price Index for Health care and social assistance with the financial year 2018/19 as the base year (ABS, 2020).[2]

Analysis was performed for the raw output (the total number of bed days) and the casemix adjusted output (the total number of bed days adjusted by relative value unit (RVU) individual casemix). The aggregated output was used (which is denoted with Y or Y^* to distinguish from y) (i.e. aggregating the number of bed days in the four bed day categories).

The quality of the data was checked and explored, in particular, for the existence of outliers and their nature to reduce the likelihood of issues arising in the regression analysis. This is important because the results may be heavily influenced by the quality of data in general and by outliers that may contaminate the sample and potentially distort the results. More details on the quality of the data and the proposed steps that were undertaken are provided in *Appendix A* of the *Main Report*. Based on the data diagnostics, the outliers that were 2.5% in each tail of the distribution of average cost (cost per casemix adjusted bed days) were removed for the regression analysis. It resulted in a sample of 5,880 observations (i.e. 308 out of 6,188 observations were dropped), which remains a relatively large sample compared to most studies in the literature. This trimmed sample represented the original sample well.

In line with the common practices of the literature as outlined in a recent systematic review (Tran et al., 2019), facility characteristics, including provider types, remoteness, size and occupancy rates, were controlled for in the analysis. Provider types were incorporated by using two dummies variables: for-profit facilities and government facilities (with not-for-profit as the base category). Remoteness was included using a remote dummy variable, representing facilities located in remote areas. Based on expert opinion, facility size was classified as either small (less than 30 places, n=318), medium or large and was used to construct a dummy variable, *small*, for inclusion in the regression analysis. *Occupancy rate* was incorporated with the unit of measurement of percentage.

The average cost per bed and average cost per casemix adjusted bed day by facility characteristics in the working sample (i.e. the trimmed sample) are presented in Table 1.

Table 1. Average cost across facility characteristics in the working sample

	Average cost per bed day				Average cost per casemix adjusted bed day			
	Mean	**SD**	**Minimum**	**Maximum**	**Mean**	**SD**	**Minimum**	**Maximum**
All Facilities	255.11	53.48	131.90	627.61	262.39	49.28	184.92	502.50
Provider type								
Nor for profit	252.44	51.41	131.90	627.61	262.52	47.11	185.25	502.50
For profit	259.54	41.94	135.77	543.60	249.84	45.58	184.92	502.12
Government	292.39	95.44	145.10	611.18	300.64	75.39	186.51	498.05
Location								
Remote	289.96	88.65	152.56	522.37	314.94	75.53	191.83	488.90
Non-remote	254.76	52.89	131.90	627.61	261.84	48.66	184.92	502.50
Size								
Small (<30 beds)	247.60	71.68	143.39	611.18	304.29	65.64	186.30	498.05
30+ beds	255.54	52.22	131.90	627.61	259.99	47.08	184.92	502.50

SD: standard deviation

[2] The Wage Price Indexes in June of 2015, 2016, 2017, 2018, and 2019 are 121.1, 124.1, 127.1, 130.6 and 134.4, respectively.

3.2 Simple linear functional form for total cost vs. output

To investigate the relationship of interest, an exploratory data analysis was first performed by conducting a simple ordinary least squares (OLS) regression, in which total cost (in $, 2018/19 constant price) was regressed on output.

Standard axioms of a classical linear regression model were assumed. However, to account for possible heteroskedasticity and possible within-cluster correlations, robust clustering standard errors (clustered by individual facility) was used. Moreover, as a sensitivity analysis, confidence intervals were computed by utilising the most recent wild clustering bootstrap approach (Roodman et al., 2019). The resulting confidence intervals were similar to the confidence intervals constructed using robust clustering standard errors, which suggests stability in the model.

The two types of outputs, the raw output and the casemix adjusted output, are outlined below.

3.2.1 Raw output (the total number of bed days)

Denoting the total cost (in $, 2018/19 constant price) as TC and the raw output (the total number of bed days) as Y, the cost function was modelled as,

$$TC = \beta_0 + \beta_1 Y + \varepsilon,$$

where, the meaning of β_0 is that of the fixed cost[3] while the meaning of β_1 is that of the variable cost of producing/delivering an additional unit of output (i.e. additional raw or unadjusted bed day) and ε is the error term (which absorbs the variation that is not explained in this model). This can be understood as the first order approximation of the unknown cost function defined above, in the direction of the most important variable, while other variables at this stage are absorbed by the error term.

The details of the estimation are shown in Table 2.[4]

Table 2. Regression results for simple linear functional forms (raw output)

Total cost	**Model 1** **OLS, linear**
Y (occupied bed days)	255.36*** (2.85)
Constant	-12135.17 (70422.07)
Observations	5880
R^2	0.8979
Adjusted R^2	0.8978
AIC	181061.17
BIC	181074.53

AIC: Akaike information criterion; BIC: Bayesian information criterion; OLS: ordinary least squares; R^2: R-squared
Clustering robust standard errors in parentheses
* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$

Thus, the estimated simple cost function (rounded to no decimals) is given by

$$\widehat{TC} = -12{,}135 + 255\,Y,$$

where, $\widehat{TC}$ stands for the expected (or fitted) total cost given a certain level of output and when the estimate of the error term ε (or the individual deviation from the estimated regression line) is set to zero (i.e. on average, for the sample).

The estimated coefficient of Y indicates that, on average, it costs about $255 to produce one additional bed day in 2018/19 constant price as the variable cost (i.e. after the fixed costs are in place). Note that the standard

[3] Here it is worth noting that the resulting estimate of the fixed cost will be from an econometric rather than an accounting perspective—as the part of the cost in the data that does not vary with the output, on average (and derived via the least-square principle) and conditional of various factors (as in the later models).
[4] All models in this report were estimated using Stata 16.

error of this estimate is quite low (and so the 95% confidence interval - CI, from $250 to $261, is quite narrow), suggesting the relatively high accuracy of this estimate (provided the accepted assumptions hold).

Importantly, the fit of the model shows that about 90% of the variation in the total cost in the sample is explained by the variation in a single variable, the total output. This is not surprising given the cost function is one of the fundamental concepts in both economic theory and in any business practice, whether for-profit or not-for-profit activity, and given the data was of relatively good quality, especially after dealing with outliers.

It should be noted that the coefficient of the constant term (which is supposed to estimate the fixed cost) is negative although it is statistically insignificant (from zero). This might be due to the fact that, although this model explains about 90% of the variation, some important characteristics might have been missed out in this simple model. As such, other key variables that can explain the total cost were investigated whilst trying to keep the model parsimonious.

To check the relevance of the linear model, nonparametric estimators (local linear least squares) were used to study the relationship between the total cost (TC) and raw output (Y).[5] The estimation results are plotted in Figure 1. It can be seen that the estimates for the simple linear regression and the nonparametric (local linear least squares) regression are quite similar for most of the observations. For the nonparametric fit, some non-linearity can be observed at the right tail of the distribution of output (in the upper 1%), but this is likely due to much greater sparsity of the data in that region than the true reflection of the shape of the relationship. Since the curvature of the non-parametric and parametric linear regressions are quite similar, for simplicity of explanations the parametric regression is described here.

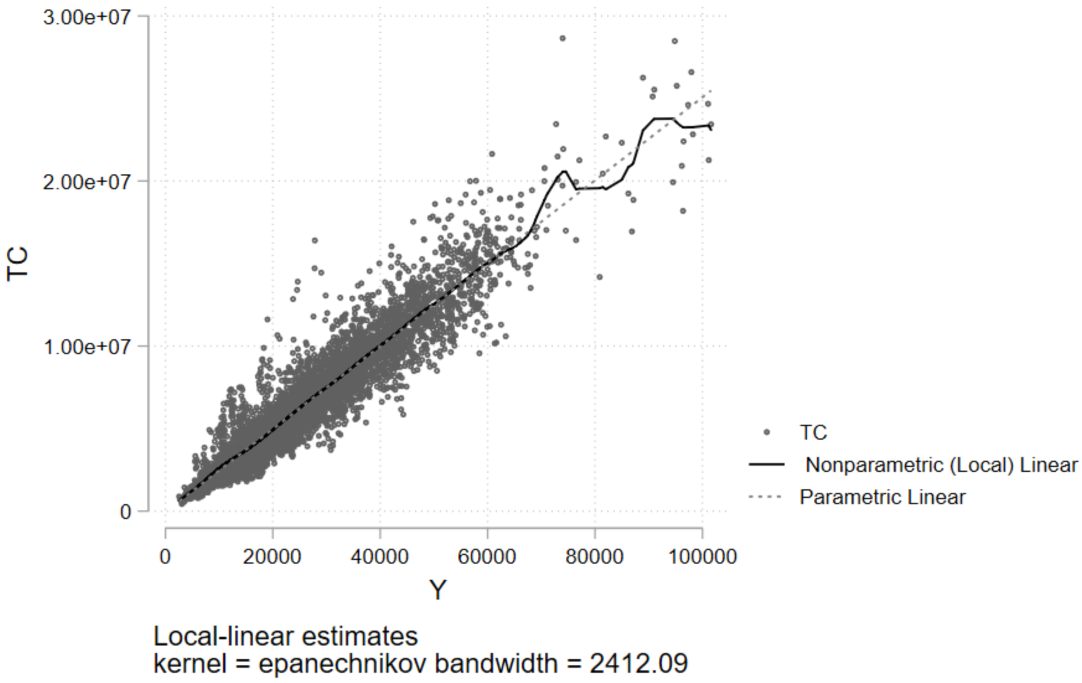

Figure 1. Nonparametric and parametric linear regression using raw output

3.2.2 Casemix adjusted output

Similar analysis as above was conducted, but with output adjusted for the complexity via the casemix index (Individual RVU) and denoted as Y^*. The details of the estimation are shown in Table 3.

[5] See detailed discussion about nonparametric estimators in Fan and Gijbels (1992, 1996), Simar et al. (2017), and Parmeter and Zelenyuk (2019).

Table 3. Regression results for simple linear functional forms (casemix adjusted output)

Total cost	Model 2 OLS, linear
Y_* (casemix adjusted occupied bed days)	232.41*** (2.11)
Constant	551464.15*** (49737.3)
Observations	5880
R^2	0.9134
Adjusted R^2	0.9134
AIC	180089.23
BIC	180102.58

AIC: Akaike information criterion; BIC: Bayesian information criterion; OLS: ordinary least squares; R^2: R-squared
Clustering robust standard errors in parentheses
* p < 0.10; ** p < 0.05; *** p < 0.01

Again, although a very simple model, the results are fairly intuitive. That is, the intercept is large and positive, while total cost is significantly positively related with output. The variation in output explains almost 91% of the variation in total cost (i.e., showing a fairly good fit of the data), despite ignoring many other characteristics of the observations (which are considered below).

The estimated relationship, can be also summarised in the following equation:

$$\widehat{TC} = 551{,}464 + 232\,Y^*$$

That is, the estimation results from this simple model suggests that, on average, it costs about $232 to produce one unit of the additional adjusted bed day, in addition to the fixed cost estimated here at $551,464, on average and in 2018/19 constant price. Note that the one unit of the adjusted bed day is the one that has the average casemix, while a more complex (than the average) bed day from the raw output data is counted here as more than one day and a less complex (than the average) bed day from the raw output data is counted here as less than one day when adjusted. This is the main reason for the difference in the estimates. In general, different casemix indexes (or its different rescaling) may produce different estimates and so should be interpreted carefully with a reference to how the casemix adjustment index was constructed and implemented.

As in the case of raw output, nonparametric estimators (local linear least squares) were also tried in order to study the relationship between the total cost (TC) and casemix adjusted output (Y^*). The estimation results are plotted in Figure 2. One can see that the estimates for the simple linear regression and the nonparametric (local linear least square) regression are very similar, even at the right tail of the distribution of output where there is greater sparsity of the data. Since, again, the curvature of the non-parametric regression is quite similar to that of the parametric linear regression, for simplicity of explanations the parametric regression was performed.

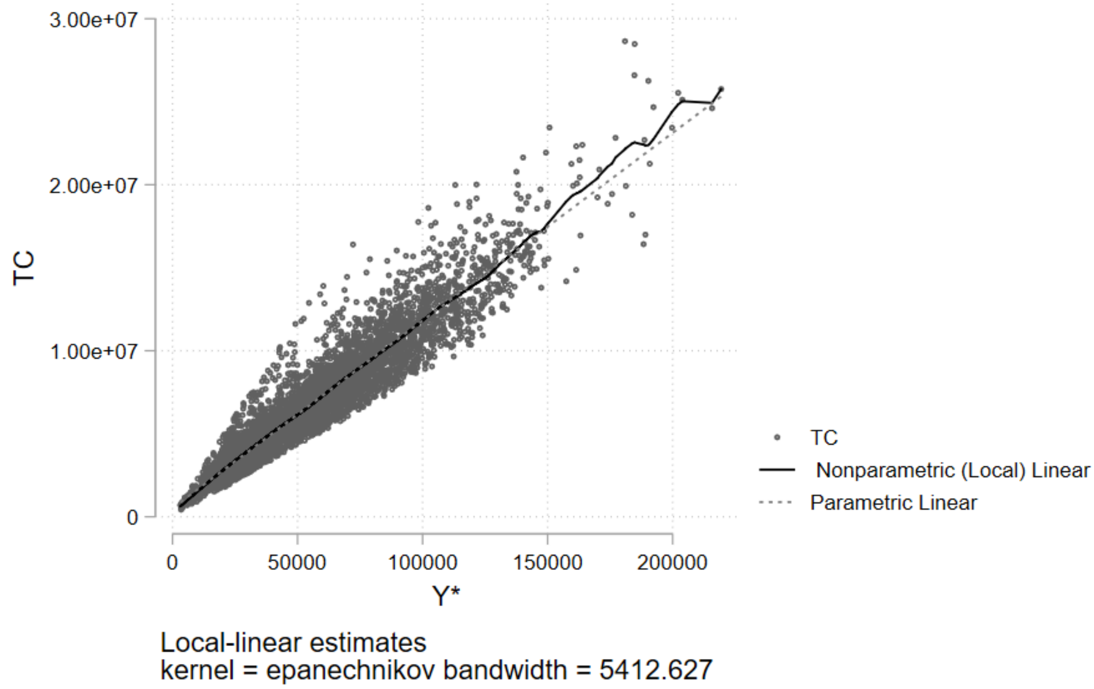

Figure 2. Nonparametric and parametric linear regression using casemix adjusted output

3.3 Conditioning on other variables

While the previous model, despite its simplicity, was found to be quite informative and was a good fit for the data, there remains scope to improve the modelling without adding significant complexity.

The influence of facility characteristics, state fixed effects and annual fixed effects in the cost function relationship were explored. Specifically, apart from the variable representing output, the regression model incorporated: year dummies (with the year 2018/19 as the base category), state dummies (with Australian Capital Territory as the base category), variables representing facility characteristics (e.g. *remote*, *for-profit*, *government*, *small*, *occupancy rate*). Moreover, the interaction between Y and each of categorical variables representing facility characteristics (e.g. *remote*, *for-profit*, *government*, *small*) was also incorporated to examine if the average variable cost was different for any of the different categories.

As before, the two types of output, the raw output and the casemix adjusted output, are detailed below.

3.3.1 Raw output (the total number of bed days)

To regress the multivariate model, three common approaches for panel data were considered, which were pooled OLS, fixed effects and random effects. The regression results for both fixed effects, random effects, pooled OLS (with robust clustering standard errors) are summarised in Table 4.

Diagnostic tests to choose the most appropriate approach among the three were performed. Specifically, the Breusch-Pagan Lagrange multiplier test indicated that the random effects approach was more appropriate than pooled OLS. Meanwhile, the F-test and the Hausman test showed that the fixed effects was more appropriate than pooled OLS and random effects approaches, respectively. As a result, the below discussion is focussed on the fixed effects model.

Moreover, a modified Wald test for *groupwise* heteroskedasticity was performed in the fixed effect regression model and evidence of *groupwise* heteroscedasticity was found. Thus, as in the case of the simple linear regression discussed above, the robust clustering standard errors was used in the fixed effects model. As a sensitivity analysis, confidence intervals were computed by utilising the wild clustering bootstrap (Roodman et al., 2019). The resulting confidence intervals were similar to the confidence intervals constructed using robust clustering standard errors, which is a good sign of stability. It should be noted, however, that the cluster-robust standard errors are known to be overly conservative, while not accounting for them usually yields standard errors that are too small. As such, a more conservative approach is taken here.

As discussed above, the focus of this discussion is on the fixed effects model with robust clustering standard errors (i.e. Model 3 in Table 4). It is worth noting that almost 99% of variation in total cost can be explained by the model, indicating a good fit of the data. This measure of goodness of fit has limitations and is based on the stated assumptions. Moreover, note that since the state dummies and variable remote are time-invariant, in the fixed effects model they are absorbed by the individual fixed effects.[6]

From the regression results, it can be seen that all the coefficients of year dummies are significantly negative and lower in magnitude for the earlier years than the latter years, indicating that, on average and ceteris paribus, the fixed cost increases across years, as expected. The coefficient of Y is statistically significant and positive, suggesting that, on average and ceteris paribus, after controlling for other explanatory factors in the estimation, it costs about $204 for a non-remote, non-small and not-for-profit facility to produce one unit of the additional bed day.

The standard error of Y is still reasonably small (although larger than in the simple model) $14 (i.e. under 10% relative to the estimate) and the estimated 95% confidence interval of this coefficient remains relatively narrow (95% CI: $179; $227).

The coefficient of constant term is statistically significant and positive, indicating that, on average and ceteris paribus, the fixed cost of a not-for-profit, non-small facility in the financial year 2018/2019 is around $2,205,918.[7] The standard error of constant term is relatively small, $326,926 (i.e. under 15% relative to the estimate) with an estimated 95% confidence interval (95% CI: $1,564,661; $2,847,174).

The significance of the estimate for the coefficient of *occupancy rate* is consistent with other studies in the literature, indicating that increasing the occupancy rate is associated with a reduction in total cost, on average and ceteris paribus. Specifically, holding other things constant, when occupancy rate increases by 1%, total cost, on average, decreases by $8,140. The standard error of this coefficient is $3,204 (i.e. around 40% of the estimate), and the estimated 95% confidence interval is relatively wide (95% CI: -$13,662; -$2,527).

While the coefficient of *for-profit* is not statistically significant, the interaction term between *for-profit* and Y is positive and statistically significant, suggesting that, on average and ceteris paribus, it costs about $51 more (in addition to $204) for for-profit facilities to produce an additional bed day compared to not-for-profit facilities. The standard error of this estimated coefficient is relatively high, about $21 (i.e. around 40% of the estimate) and the estimated 95% confidence interval is fairly wide (95% CI: $12; $87). The results suggest that, on average and ceteris paribus, there is no significant difference in fixed costs between for-profit and not-for-profit facilities, but for-profit facilities tend to produce additional bed days with more variable costs compared to not-for-profit facilities.

[6] Variable *small* is time-variant since the number of places of facilities can change across time. Similarly, *government* and not-for-profit are also time-variant since the ownership of facilities can change across time.

[7] It is worth mentioning that the constant term here is the average of individual fixed effects, and because state dummies and variable *remote* are absorbed by the individual fixed effects, the estimated fixed cost is averaged across categories defined by these variables and possibly is too large for some facilities while too small for others. If requested, the individual fixed effects can be estimated for each facility.

Table 4. Regression results for multiplicative linear models (raw output)

Total cost	**Model 3 Fixed effects, linear (clustering s.e.)**	**Model 4 Random effects, linear (clustering s.e.)**	**Mode 5 Pooled OLS, linear (clustering s.e.)**
Y (occupied bed days)	203.76***	239.37***	255.07***
	(13.58)	(4.35)	(3.42)
Financial year 2014/15	-496516.95***	-473401.20***	-522304.53***
	(32794.11)	(28360.64)	(35844.56)
Financial year 2015/16	-377048.77***	-365034.50***	-421863.24***
	(29061.07)	(24618.34)	(32501.02)
Financial year 2016/17	-248880.77***	-240214.97***	-288994.94***
	(25086.71)	(21403.29)	(28972.36)
Financial year 2017/18	-143018.22***	-135511.22***	-177777.10***
	(22083.04)	(18915.28)	(25948.02)
New South Wales	-	-128260.74	-110044.50
		(289049.70)	(277039.24)
Northern Territory	-	559392.52*	587865.48**
		(286458.52)	(273747.66)
Queensland	-	249916.37	301941.18
		(295311.60)	(283178.83)
South Australia	-	93729.14	114639.55
		(295883.88)	(281899.14)
Tasmania	-	314315.97	312517.45
		(310661.67)	(294854.84)
Victoria	-	222217.62	294113.50
		(290561.37)	(277562.13)
Western Australia	-	136169.77	135390.28
		(298561.05)	(287030.22)
Remote	-	392729.43	905348.19**
		(323173.36)	(408946.49)
For-profit	-387236.60	-635009.82***	-454062.93**
	(584305.03)	(230906.02)	(178145.38)
Government	-218303.30	-88247.26	56357.53
	(515713.62)	(227348.98)	(265701.67)
Small	17220.27	-335920.19*	-47440.67
	(326185.72)	(185857.56)	(197984.52)
Occupancy rate	-8140.00**	-13447.75***	-16656.66***
	(3204.07)	(1996.41)	(2164.86)
Y x Remote	41.46	-17.69	-37.64
	(75.17)	(33.66)	(34.47)
Y x For-profit	51.19**	27.19***	16.32**
	(20.89)	(8.56)	(6.80)
Y x Government	36.13	30.80**	29.83
	(29.28)	(15.55)	(18.19)
Y x Small	-3.33	23.71	0.12
	(23.18)	(16.01)	(24.95)
Constant	2205917.58***	1825722.66***	1706780.29***
	(326925.62)	(336629.30)	(330493.33)
Observations	5880	5880	5880
R^2	0.9878		0.9062
Adjusted R^2	0.9834		0.9058
AIC	168569.14	.	180599.38
BIC	168655.97	.	180739.65

AIC: Akaike information criterion; BIC: Bayesian information criterion; OLS: ordinary least squares; R^2: R-squared; s.e.: standard error.
The base case is: financial year 2018/19, Australian Capital Territory, not remote, not-for-profit provider, 30+ beds.
Clustering robust standard errors in parentheses
* p < 0.10; ** p < 0.05; *** p < 0.01

Regarding other facility characteristics, no statistical evidence about their association with the difference in the cost structure of facilities was found. This, however, does not imply there is no difference, especially because the cluster robust standard errors are known to often be too conservative (i.e. too large) and this could be the reason for the observed statistical insignificance. On the other hand, the robust standard errors without accounting for the clusters are usually too small (although in some cases can also be more accurate than when accounting for the clusters).[8]

[8] See Abadie et al. (2017) and McKinnon (2019) for the most recent discussions on the caveats in accounting for clusters.

3.3.2 The case of casemix adjusted output

As is the case for the raw output, diagnostic tests were performed to choose the appropriate approach to estimate the model and make statistical inferences. The test results also showed that the appropriate approach is fixed effects with clustering robust standard error (i.e. the F-test and the Hausman test showed that fixed effects is more appropriate than pooled OLS and random effects, respectively; the Breusch-Pagan Lagrange multiplier test indicated that random effects is more appropriate than pooled OLS; and modified Wald test showed evidence of groupwise heteroskedasticity). Confidence intervals were computed using the wild clustering bootstrap approach.

The regression results for both fixed effects, random effects, pooled OLS (with robust clustering standard errors) are summarised in Table 5. The below discussion is focussed on Model 6 (i.e. fixed effects with robust clustering standard error). Again, almost 99% of variation in total cost can be explained by the model, indicating a good fit of the data.

All the coefficients of year dummies are significantly negative and lower in magnitude for the earlier years than latter years, indicating that, on average and ceteris paribus, the fixed cost increases across years. The coefficient of Y^* is statistically significant and positive, suggesting that, on average and ceteris paribus, after controlling for other confounding factors, it costs about $140 for a non-remote, non-small and not-for-profit facility to produce one unit of the additional casemix adjusted bed day. Also note that the standard error of Y^* is reasonably small, $10 (i.e. under 10% relative to the estimate), and the estimated 95% confidence interval of this coefficient is relatively narrow (95% CI: $122; $159).

The coefficient of constant term is statistically significant and positive, indicating that, on average and ceteris paribus, the fixed cost of a not-for-profit, non-small facility in the financial year 2018/2019 is around $2,618,631.[9] The standard error of constant term is fairly small, $380,197 (i.e. under 15% relative to the estimate), resulting in a fairly narrow estimated 95% confidence interval (95% CI: $1,872,884; $3,364,378).

While the coefficient of *for-profit* is not statistically significant, the interaction term between *for-profit* and Y^* is significantly positive (at 5% level of significance), suggesting that, on average and ceteris paribus, it costs about $45 more (in addition to $140) for for-profit facilities to produce an additional casemix adjusted bed day compared to not-for-profit facilities. The standard error of this estimated coefficient is relatively high, $22 (i.e. around 50% of the estimate) and the estimated 95% confidence interval is fairly wide (95% CI: $2; $83). The results suggest that, on average and ceteris paribus, there is no significant difference in fixed costs between for-profit and not-for-profit facilities, but for-profit facilities tend to produce additional casemix adjusted bed days with more variable costs compared to not-for-profit facilities. Regarding other facility characteristics, no statistical evidence about their association with the difference in the cost structure of facilities were found.

[9] Again, it is worth reminding that the constant term here is the average of individual fixed effects, and because state dummies and variable *remote* are absorbed by the individual fixed effects, the estimated fixed cost is averaged across categories defined by these variables.

Table 5. Regression results for multiplicative linear models (casemix adjusted output)

Total cost	Model 6 Fixed effects, linear (clustering s.e.)	Model 7 Random effects, linear (clustering s.e.)	Mode 8 Pooled OLS, linear (clustering s.e.)
Y^* (casemix adjusted ccupied bed days)	140.25***	205.55***	232.26***
	(10.33)	(3.04)	(2.54)
Financial year 2014/15	-344968.28***	-201084.89***	-177002.85***
	(36336.37)	(31673.94)	(38042.38)
Financial year 2015/16	-302138.50***	-228647.30***	-231536.99***
	(31504.33)	(28298.37)	(35409.71)
Financial year 2016/17	-161079.48***	-94073.73***	-79971.90**
	(26995.80)	(24798.10)	(31690.36)
Financial year 2017/18	-95187.90***	-51483.97**	-32827.35
	(23740.49)	(21547.06)	(27450.09)
New South Wales	-	-432823.12*	-320822.23
		(243219.08)	(223902.14)
Northern Territory	-	181353.18	264081.24
		(239857.30)	(220835.92)
Queensland	-	-350945.17	-345951.02
		(249716.08)	(230486.12)
South Australia	-	-335905.01	-225651.43
		(252195.07)	(230571.51)
Tasmania	-	40520.69	58572.53
		(287560.72)	(259738.67)
Victoria	-	-196915.97	-35381.46
		(246795.38)	(227068.81)
Western Australia	-	-457862.02*	-365265.75
		(248445.04)	(229225.86)
Remote	-	527589.27	491956.40
		(366748.23)	(392882.39)
For profit	117391.57	-710085.32***	-547263.12***
	(668290.03)	(219479.85)	(153433.13)
Government	273094.81	-195262.25	-193366.12
	(455431.59)	(204010.80)	(239385.08)
Small	184295.33	-584052.38***	-261180.70*
	(288604.83)	(189918.65)	(139029.33)
Occupancy rate	4172.58	-6606.37***	-12672.92***
	(3431.31)	(2084.43)	(2157.48)
Y^* x Remote	-70.47	-33.24	-8.31
	(45.48)	(31.77)	(36.32)
Y^* x For-profit	44.51**	18.65**	6.50
	(21.90)	(7.92)	(5.79)
Y^* x Government	2.40	23.30	31.17*
	(42.06)	(15.33)	(18.39)
Y^* x Small	-50.48	14.70	10.21
	(39.04)	(29.77)	(23.02)
Constant	2618631.37***	2379646.96***	2126802.12***
	(380196.99)	(311429.05)	(295530.36)
Observations	5880	5880	5880
R^2	0.9856		0.9177
Adjusted R^2	0.9803		0.9174
AIC	169569.55	.	179828.65
BIC	169656.38	.	179968.91

AIC: Akaike information criterion; BIC: Bayesian information criterion; OLS: ordinary least squares; R^2: R-squared; s.e.: standard error.
The base case is: financial year 2018/19, Australian Capital Territory, not remote, not-for-profit provider, 30+ beds.
Clustering robust standard errors in parentheses
* p < 0.10; ** p < 0.05; *** p < 0.01

3.4 Higher order polynomials of output

In addition to the nonparametric analysis of the shape of the cost function, the effect of the square and cubic terms for the output variables ($Y2$ and $Y3$ for raw output, and $Y2^*$ and $Y3^*$ for casemix adjusted output) were explored to determine if there is significant non-linearity in the relationship (which can be captured by up to third order polynomial). The results presented in Table 6 and Table 7 are consistent with our earlier conclusion from the nonparametric analysis. Indeed, the estimated coefficients for the output square and output cubic terms in the models are not statistically significant (and not jointly significant, according to the F-test), and their magnitudes are extremely small and do not add much, or at all, to the goodness-of-fit of the model.

Table 6. Regression results for higher order polynomials of output (raw output)

Total cost	**Model 9** **OLS, higher order polynomial**
Y (occupied bed days)	207.9***
	(64.34)
Y^2	0.0000188
	(0.00183)
Y^3	-1.16e-09
	(1.54e-08)
Financial year 2014/15	-495672.8***
	(32150.9)
Financial year 2015/16	-376574.7***
	(28783.6)
Financial year 2016/17	-248755.6***
	(24940.3)
Financial year 2017/18	-142926.0***
	(22004.6)
For profit	-381266.8
	(594411.4)
Government	-177586.7
	(545804.8)
Small	24281.6
	(341392.4)
Occupancy rate	-8357.4**
	(3267.2)
Y x Remote	38.99
	(75.85)
Y x For-profit	50.94**
	(21.26)
Y x Government	33.33
	(31.67)
Y x Small	-3.474
	(24.41)
Constant	2140410.6***
	(619018.8)
Observations	5880
R^2	0.9878
Adjusted R^2	0.9834
AIC	168571.31
BIC	168671.50

AIC: Akaike information criterion; BIC: Bayesian information criterion; OLS: ordinary least squares; R^2: R-squared; s.e.: standard error; The base case is: financial year 2018/19, Australian Capital Territory, not remote, not-for-profit provider, 30+ beds.
Clustering robust standard errors in parentheses
* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$

Table 7. Regression results for higher order polynomials of output (casemix adjusted output)

Total cost	**Model 10** **OLS, high order polynomial**
Y* (casemix adjusted occupied bed days)	162.7***
	(36.17)
Y^{*2}	-0.000723
	(0.000907)
Y^{*3}	6.24e-09
	(6.88e-09)
Financial year 2014/15	-346446.0***
	(37754.4)
Financial year 2015/16	-303608.3***
	(32511.5)
Financial year 2016/17	-161183.8***
	(27169.4)
Financial year 2017/18	-96072.0***
	(23915.0)
For-profit	117638.1
	(665383.2)
Government	352913.2
	(484753.2)
Small	261340.6
	(313587.8)
Occupancy rate	4093.0
	(3420.7)
Y* x Remote	-71.64
	(45.21)
Y* x For-profit	44.44**
	(21.88)
Y* x Government	-1.937
	(43.39)
Y* x Small	-58.40
	(42.34)
Constant	2450705.4***
	(502543.9)
Observations	5880
R^2	0.9856
Adjusted R^2	0.9803
AIC	169562.84
BIC	169663.03

AIC: Akaike information criterion; BIC: Bayesian information criterion; OLS: ordinary least squares; R^2: R-squared.
The base case is: financial year 2018/19, Australian Capital Territory, not remote, not-for-profit provider, 30+ beds.
Clustering robust standard errors in parentheses
* p < 0.10; ** p < 0.05; *** p < 0.01

3.5 Log-log specification

In the literature, the log-log specification for the cost function estimation is quite a common practice. The logarithmic functional form helps to partially take care of the heteroskedasticity in the data. Moreover, the coefficient corresponding to output in the log-log specification has a convenient interpretation, which is the elasticity of total cost with respect to output.[10] In this section, similar analysis are presented as in the Section 3.3 with the logarithmic functional form of total cost and output.

As before, the two types of output, the raw output and the casemix adjusted output, are detailed below.

3.5.1 Raw input (the total number of bed days)

As with the case of linear functional form, diagnostic tests were performed to choose the appropriate approach to estimate the model and make statistical inferences. The test results also showed that the appropriate approach is fixed effects with clustering robust standard error (i.e. the F-test and the Hausman test showed that fixed effects is more appropriate than pooled OLS and random effects, respectively; the Breusch-Pagan Lagrange multiplier test indicated that random effects is more appropriate than pooled OLS; and modified Wald

[10] This might be sometimes also viewed as restrictive because it imposes the assumption of constancy of elasticity of total cost with respect to output.

test showed evidence of groupwise heteroskedasticity). Confidence intervals were computed using the wild clustering bootstrap approach.

As discussed above, here we focus our discussion on a fixed effects model with robust clustering standard errors (i.e. Model 11 in Table 8). Again, almost 99% of variation in total cost can be explained by the model, indicating a good fit of the data.

From the regression results, it can be seen that all the coefficients of year dummies are significantly negative and lower in magnitude for the earlier years than the latter years, indicating that, on average and ceteris paribus, the fixed cost increases across years, as expected. The coefficient of lnY is statistically significant and positive, suggesting that, on average and ceteris paribus, after controlling for other explanatory factors in the estimation, when bed days increase by 1%, the total cost increases by about 0.77% for a non-remote, non-small and not-for-profit facility. The standard error of lnY is still reasonably small 0.04 (i.e. around 5% relative to the estimate), and the estimated 95% confidence interval of this coefficient remains relatively narrow (95% CI: 0.70; 0.84).

The coefficient of *government* is significantly negative (at 10% level of significance) suggesting that the fixed cost of government facilities is, on average and ceteris paribus, 262% less than that of a not-for-profit facility. The standard error of this estimate is relatively high at 1.52 (i.e. more than 50% of the estimate), and the corresponding confidence interval is relatively wide (95% CI: -7.00; 0.57). Meanwhile, the interaction term between *government* and lnY is significantly positive (at 10% level of significance) indicating that the elasticity of total cost with respect to output is 0.28 more (in addition to 0.77) for government facilities compared to not-for-profit facilities, on average and ceteris paribus. The standard error of the interaction term is relatively high at 0.16 (i.e. more than 50% of the estimate), resulting in a relatively wide confidence interval (95% CI: -0.07; 0.74).

The estimation coefficient of *occupancy rate* is significantly negative, indicating that, on average and ceteris paribus, when occupancy rate increases by 1%, total cost decreases by 0.2%. The standard error of this coefficient is relatively small at 0.0004 (i.e. around 20% of the estimate), and the estimated 95% confidence interval is relatively narrow (95% CI: -0.0025; -0.0009).

Table 8. Regression results for multiplicative models with log-log specification (raw output)

lnTC (log of total cost)	Model 11 Fixed effects, log log (clustering s.e.)	Model 12 Random effects, log log (clustering s.e.)	Mode 13 Pooled OLS, log log (clustering s.e.)
lnY (log of occupied bed days)	0.766***	0.940***	1.026***
	(0.04)	(0.02)	(0.01)
Financial year 2014/15	-0.085***	-0.081***	-0.090***
	(0.00)	(0.00)	(0.01)
Financial year 2015/16	-0.062***	-0.060***	-0.070***
	(0.00)	(0.00)	(0.01)
Financial year 2016/17	-0.039***	-0.037***	-0.045***
	(0.00)	(0.00)	(0.00)
Financial year 2017/18	-0.021***	-0.019***	-0.026***
	(0.00)	(0.00)	(0.00)
New South Wales	-	-0.053*	-0.042
		(0.03)	(0.03)
Northern Territory	-	0.083***	0.084***
		(0.03)	(0.03)
Queensland	-	0.020	0.039
		(0.03)	(0.03)
South Australia	-	-0.001	0.010
		(0.03)	(0.03)
Tasmania	-	0.044	0.048
		(0.03)	(0.03)
Victoria	-	0.020	0.039
		(0.03)	(0.03)
Western Australia	-	0.004	0.007
		(0.03)	(0.03)
Remote	-	0.968	1.059
		(0.74)	(0.85)
For-profit	-1.284	-0.399	-0.039
	(0.90)	(0.39)	(0.26)
Government	-2.621*	0.070	1.055**
	(1.52)	(0.59)	(0.53)
Small	0.503	-0.567	-0.033
	(0.83)	(0.51)	(0.58)
Occupancy rate	-0.002***	-0.003***	-0.003***
	(0.0004)	(0.0003)	(0.0004)
lnY x Remote	0.067	-0.095	-0.100
	(0.20)	(0.08)	(0.09)
lnY x For-profit	0.142	0.042	0.004
	(0.09)	(0.04)	(0.02)
lnY x Government	0.284*	0.002	-0.098*
	(0.16)	(0.06)	(0.06)
lnY x Small	-0.060	0.054	-0.001
	(0.09)	(0.06)	(0.07)
Constant	8.042***	6.429***	5.600***
	(0.37)	(0.15)	(0.12)
Observations	5880	5880	5880
R^2	0.9898		0.9051
Adjusted R^2	0.9861		0.9048
AIC	-16695.92	.	-3554.72
BIC	-16609.09	.	-3414.45

AIC: Akaike information criterion; BIC: Bayesian information criterion; OLS: ordinary least squares; R2: R-squared; s.e.: standard error.
The base case is: financial year 2018/19, Australian Capital Territory, not remote, not-for-profit provider, 30+ beds.
Clustering robust standard errors in parentheses
* p < 0.10; ** p < 0.05; *** p < 0.01

3.5.2 The case of casemix adjusted output

As is the case with the raw output, diagnostic tests were performed to choose the appropriate approach to estimate the model and make statistical inferences. The test results also showed that the appropriate approach is fixed effects with clustering robust standard error (i.e. the F-test and the Hausman test showed that fixed effects is more appropriate than pooled OLS and random effects, respectively; the Breusch-Pagan Lagrange multiplier test indicated that random effects is more appropriate than pooled OLS; and modified Wald test showed evidence of groupwise heteroskedasticity). Confidence intervals were computed using the wild clustering bootstrap approach.

The regression results for both fixed effects, random effects, pooled OLS (with robust clustering standard errors) are summarised in Table 9. The below discussion is focussed on Model 14 (i.e. fixed effects with robust clustering standard error). Again, almost 99% of variation in total cost can be explained by the model, indicating a good fit of the data.

From the regression results, it can be seen that all the coefficients of year dummies are significantly negative and lower in magnitude for the earlier years than the latter years, indicating that, on average and ceteris paribus, the fixed cost increases across years, as expected. The coefficient of lnY^* is statistically significant and positive, suggesting that, on average and ceteris paribus, after controlling for other explanatory factors in the estimation, when casemix adjusted bed days increase by 1%, the total cost increases by about 0.52% for a non-remote, non-small and not-for-profit facility. The standard error of lnY^* is still reasonably small 0.03 (i.e. around 5% relative to the estimate), and the estimated 95% confidence interval of this coefficient remains relatively narrow (95% CI: 0.46; 0.58).

The coefficient of *for-profit* is significantly negative (at 10% level of significance) suggesting that the fixed cost of for-profit facility is, on average and ceteris paribus, 144% less than that of a not-for-profit facility. The standard error of this estimate is relatively high at 0.81 (i.e. more than 50% of the estimate), and the corresponding confidence interval is relatively wide (95% CI: -2.98; 0.34). Meanwhile, the interaction term between *for-profit* and lnY^* is significantly positive (at 10% level of significance) indicating that the elasticity of total cost with respect to output is 0.16 more (in addition to 0.52) for for-profit facilities compared to not-for-profit facilities, on average and ceteris paribus. The standard error of the interaction term is relatively high at 0.08 (i.e. around 50% of the estimate), resulting in a relatively wide confidence interval (95% CI: -0.02; 0.31).

The coefficient of *small* is significantly positive suggesting that the fixed cost of a small facility is, on average and ceteris paribus, 126% more than that of a non-small facility. The standard error of this estimate is relatively high at 0.68 (i.e. more than 50% of the estimate), and the corresponding confidence interval is relatively wide (95% CI: 0.01; 2.58). Meanwhile, the interaction term between *small* and lnY^* is significantly negative (at 5% level of significance) indicating that the elasticity of total cost with respect to output is 0.15 less for small facilities compared to non-small facilities, on average and ceteris paribus. The standard error of the interaction term is relatively high at 0.08 (i.e. around 50% of the estimate), resulting in a relatively wide confidence interval (95% CI: -0.30; -0.01).

Table 9. Regression results for multiplicative linear models (casemix adjusted output)

lnTC (log of total cost)	Model 14 Fixed effects, log log (clustering s.e.)	Model 15 Random effects, log-log (clustering s.e.)	Mode 16 Pooled OLS, log log (clustering s.e.)
lnY^* (log of casemix adjusted occupied bed days)	0.521***	0.797***	0.898***
	(0.033)	(0.009)	(0.007)
Financial year 2014/15	-0.053***	-0.026***	-0.022***
	(0.005)	(0.004)	(0.005)
Financial year 2015/16	-0.044***	-0.030***	-0.030***
	(0.004)	(0.004)	(0.005)
Financial year 2016/17	-0.021***	-0.008**	-0.004
	(0.004)	(0.003)	(0.004)
Financial year 2017/18	-0.012***	-0.004	0.001
	(0.003)	(0.003)	(0.004)
New South Wales	-	-0.065**	-0.043
		(0.031)	(0.027)
Northern Territory	-	0.039	0.040
		(0.030)	(0.026)
Queensland	-	-0.055*	-0.050*
		(0.032)	(0.028)
South Australia	-	-0.060*	-0.037
		(0.032)	(0.028)
Tasmania	-	0.007	0.007
		(0.036)	(0.032)
Victoria	-	-0.019	0.010
		(0.031)	(0.027)
Western Australia	-	-0.053*	-0.036
		(0.032)	(0.028)
Remote	-	0.427	0.224
		(0.505)	(0.515)
For-profit	-1.442*	-0.606**	-0.430**
	(0.809)	(0.237)	(0.169)
Government	-0.913	0.251	0.075
	(1.443)	(0.460)	(0.327)
Small	1.257**	0.668	0.967***
	(0.676)	(0.472)	(0.374)
Occupancy rate	0.0003	-0.002***	-0.002***
	(0.0005)	(0.0003)	(0.0004)
lnY^* x Remote	-0.100	-0.041	-0.014
	(0.126)	(0.055)	(0.057)
lnY^* x For-profit	0.161*	0.057**	0.036**
	(0.080)	(0.023)	(0.017)
lnY^* x Government	0.116	-0.022	-0.003
	(0.161)	(0.048)	(0.035)
lnY^* x Small	-0.151**	-0.090*	-0.113***
	(0.077)	(0.055)	(0.043)
Constant	10.339***	7.805***	6.827***
	(0.314)	(0.091)	(0.081)
Observations	5880	5880	5880
R^2	0.9884		0.9322
Adjusted R^2	0.9841		0.9320
AIC	-15918.54	.	-5530.30
BIC	-15831.71	.	-5390.03

AIC: Akaike information criterion; BIC: Bayesian information criterion; OLS: ordinary least squares; R2: R-squared; s.e.: standard error.
The base case is: financial year 2018/19, Australian Capital Territory, not remote, not-for-profit provider, 30+ beds.
Clustering robust standard errors in parentheses
* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$

3.6 Other explanatory variables and caveats

While many other explanatory variables (relative to the simplest model) that were considered important from the perspective of experts in the field did not show statistical significance,[11] this does not mean those variables are not important and can be dropped without any loss of information. Indeed, if the explanatory variables are relevant from the economics of aged care point of view and statistically significant, their omission may (and in

[11] We also explored Machine Learning approaches based on Lasso and confirmed the relatively greater significance of such variables as output, occupancy rate, and facility types among all the explanatory variables we have considered in this study.

some cases do) introduce estimation bias to the estimates of other coefficients. This same caveat, of course, applies to any other variables that might have been missed (e.g. due to data availability). From economic theory, it is known that the cost function also typically depends on prices of each input, as is also seen in the definition of the cost function above.[12] Unfortunately, the data on the prices paid to every input (hundreds of them, potentially) by each facility in each year was unavailable at the time of this analysis. As a result, some of the variation (if present) in the paid prices for all the inputs is essentially left to be in the unexplained variation in the total cost (which, fortunately, is relatively low) and, potentially, might have to some extent impacted the magnitude of all the other coefficients. In the absence of additional data, Models 3, 6, 11 and 14 (or its slightly simpler version) is considered most appropriate and these were leveraged in the development of the frontier models.

[12] E.g., see Sickles and Zelenyuk (2019, Chapter 2 and 11).

4. Stochastic frontier modelling for total cost

4.1 Theoretical framework

In this technical supplementary report, an econometric modelling approach known as stochastic frontier analysis is applied to estimate the efficiency of residential aged care facilities. A myriad of stochastic frontier analysis models have been suggested in the econometric literature, each having their advantages and limitations. One of the challenges is to select a model to rely upon in any given context.[13]

Based on a preliminary regression analysis, it was concluded that the fixed effects model for panel data appears to be most suitable for our total cost data if willing to ignore potential inefficiency and just estimate the average tendency for the total cost function. Therefore, it is expected that a similar approach is among the most suitable for the frontier analysis when the focus is to account for and estimate the observed inefficiencies of residential aged care facilities. This is because inefficiency is modelled in the stochastic frontier analysis through further decomposition of the error terms, into a symmetric statistical noise and inefficiency. Thus, it is also expected that a fixed effects stochastic frontier analysis model would also be most appropriate as well, and there are several to choose from. However, because there are no strong *a priori* beliefs regarding which model is better or worse and with the goal of arrive at more robust conclusions, it is advisable to still try other relevant variations of stochastic frontier models for the same data to examine the sensitivity/robustness of the results.

To facilitate the discussion and for readers' convenience, a very brief summary of formulation of the most common stochastic frontier models in the literature (which will also be applied to study the efficiency of residential aged care facilities) is provided in this section.[14]

4.1.1 Aigner, Lovell and Schmidt (1977) (hereafter ALS77)

The pioneering and now classic model is formulated (for each observation, $i = 1, \dots, n$) as follows,

$$TC_i = \alpha + Y_i\beta_Y + Z_i'\beta_Z + \varepsilon_i, \quad i = 1, \dots, n, \tag{1}$$

$$\varepsilon_i = v_i + u_i, \tag{2}$$

$$v_i \sim_{\text{iid}} \mathcal{N}(0, \sigma_v^2), \tag{3}$$

$$u_i \sim_{\text{iid}} \mathcal{N}^+(0, \sigma_u^2), \tag{4}$$

where TC_i is the total cost, Y_i is output, and Z_i is a vector of control variables.[15] [16] The composed error term ε_i is the sum of a normally distributed disturbance, v_i, representing measurement and specification error, and a positive disturbance u_i (followed half-normal distribution), representing cost inefficiency. It is assumed that v_i and u_i are statistically independent from each other and from Y_i and Z_i, and the corresponding likelihood function is constructed and the model is then estimated using maximum likelihood estimation.

Intuitively, note that the model discussed here is a generalisation of our 'pooled OLS regression' model discussed in the preliminary regression analysis, which now attempts to also model possible inefficiency (e.g. note that for a balanced panel data that would mean, $i = 1, \dots, n = N \times T$).

Once the parameters of the stochastic frontier model have been estimated, one can obtain the <u>expected level of industry inefficiency</u> by estimating

$$E[u] = \sqrt{2/\pi}\sigma_u. \tag{5}$$

In the case of linear functional form, $E[u]$ provides information about expected industry inefficiency in absolute terms (i.e. in the same unit of measurement with the total cost). Meanwhile, for the log-log specification, $E[u]$

[13] Detailed discussions about SFA models can be found in Kumbhakar and Lovell (2000), Greene (2008), Parmeter and Kumbhakar (2014), Kumbhakar, Wang, and Horncastle (2015), Kumbhakar, Parmeter and Zelenyuk (2018) and, perhaps most comprehensively and in textbook style discussion in Sickles and Zelenyuk (2019).

[14] More details can be found in the references listed in footnote 13. Note that most of the description in cited sources is made in terms of production function rather than the cost function and so presenting the latter here would be useful.

[15] In the analysis, both raw output and casemix adjusted output will be used. Moreover, both linear and logarithmic functional forms of total cost and output will be utilised. To avoid unnecessary repetition, only formulas for linear functional form are presented, separate discussion for logarithmic functional form will be provided where necessary.

[16] It is worth mentioning here that in the original formulation, ALS77 model did not include control variables but it is natural to include them as suggested by our preliminary regression analysis.

measures the expected industry inefficiency in relative terms (i.e. in percentages). Moreover, with the logarithmic specification, the relative industry efficiency can be obtained as

$$E[exp(-u)] \approx 1 - E[u]. \tag{6}$$

Since both efficiency and inefficiency will be presented, it is useful to keep in mind the approximate relationship between these two terms described in equation (6). Also note that the above measures provide information about the average level of (in)efficiency for the sample representing a population (e.g. industry) rather than individual efficiency scores.

If interested in the estimates of individual (in)efficiency scores of a specific residential aged care facility, more elaboration is needed. The most popular approach in the literature is to follow Jondrow et al. (1982) (hereafter JLMS), where the inefficiency of a residential aged care facility, say RACFi, can be estimated or predicted using the expected value of u_i conditional on the realisation of the composed error of the model. That is, $E(u_i|\, \varepsilon_i)$, given by

$$E(u_i|\, \varepsilon_i) = \frac{\sigma_* \phi\left(\frac{\mu_{*i}}{\sigma_*}\right)}{\Phi\left(\frac{\mu_{*i}}{\sigma_*}\right)} + \mu_{*i}, \tag{7}$$

where,

$$\mu_{*i} = \frac{\sigma_u^2 \varepsilon_i}{\sigma_v^2 + \sigma_u^2}, \tag{8}$$

and

$$\sigma_* = \frac{\sigma_v^2 \sigma_u^2}{\sigma_v^2 + \sigma_u^2}, \tag{9}$$

while $\phi(\cdot)$ and $\Phi(\cdot)$ are pdf and cdf of the standard normal distribution, respectively.

It is worth noting here that $E(u_i|\, \varepsilon_i)$ measures residential aged care facility specific inefficiency in the same units as the dependent variable. That is, it would be in absolute terms (e.g. dollars) for linear functional form, and in relative terms for logarithmic functional form. Moreover, for the logarithmic specification, a convenient measure of relative efficiency $E[exp(-u_i)|\varepsilon_i]$ can also be estimated. Specifically, and similarly as for the industry average inefficiency in equation (6), the expected individual efficiency can be estimated as $E[exp(-u_i)|\varepsilon_i]$ by using the approach discussed in Battese and Coelli (1988) or utilising the following approximation

$$E[exp(-u_i)|\varepsilon_i] \approx 1 - E(u_i|\varepsilon_i). \tag{10}$$

We also note that while originally developed for ALS77, the same JLMS procedure can be applied to predict (in)efficiency of a specific residential aged care facility in the other models estimated by maximum likelihood estimation which will be discussed in the following subsections.

In brief, the main advantage of the ALS77 model is that it allows for including both inefficiency and statistical errors in the model (and so the inefficiency can be distinguished and disentangled from the statistical errors, under certain assumptions). Meanwhile, one of its limitation is that, besides the parametric assumptions on the functional form for the cost frontier, the parametric distributional assumptions are also required for the two error components (inefficiency and noise) to estimate the model and to estimate or predict the overall and individual (in)efficiency. Other distributional assumptions can be used and there is usually no strong preference over which distributions should be used in practice except, largely, for computational convenience the so called "normal-half-normal" specification presented above appears to be the most popular. Moreover, various studies in the literature confirmed the results are not very sensitive to difference distributional specifications.[17]

Finally, since the ALS77 model is a generalisation of our 'pooled OLS regression' model discussed in the preliminary regression analysis, it shares the same limitation as the pooled regression. That is, it does not account for the panel nature of the data (except for the annual effects), which is useful to account for.

[17] For the related discussion and references, see Kumbhakar, Parmeter and Zelenyuk (2018) and Sickles and Zelenyuk (2019).

4.1.2 Battese and Coelli (1995) (hereafter BC95)

The formulation of the BC95 model is similar to that of the ALS77 model, but now u_i is a non-negative truncation of a normal distribution with the pre-truncated mean being a function of some exogenous determinants. Specifically, the model is formulated as follows

$$TC_i = \alpha + Y_i \beta_Y + Z_i' \beta_Z + \varepsilon_i, \quad i = 1, \dots, n, \tag{11}$$

$$\varepsilon_i = v_i + u_i, \tag{12}$$

$$v_i \sim \mathcal{N}(0, \sigma_v^2), \tag{13}$$

$$u_i \sim \mathcal{N}^+(\mu_i, \sigma_u^2), \tag{14}$$

$$\mu_i = W_i' \, \delta, \tag{15}$$

where, W_i is a vector of exogenous determinants of inefficiency.

This is a generalisation of the ALS77 model, where its advantage is that it allows for modelling the association of exogenous determinants on inefficiency. However, as for the ALS77 model, the limitation of this model is that parametric distributional assumptions on error components are required to identify inefficiency from statistical errors (besides the functional form of the cost frontier). Moreover, a particular parametric form for the relationship between inefficiency and covariates is also required. Furthermore, it is important to acknowledge that the BC95 model often suffers from computational problems and numerical identification issues. Also note that while originally cast in panel data setup, it is not accounting for panel data heterogeneity in a way fixed effects or random effects models do and is essentially a generalisation of the pooled regression (thus sharing its limitations) with the inefficiency term being modelled on some covariates.

4.1.3 Caudill, Ford, and Gropper (1995) (hereafter CFG95)

The formulation of the CFG95 model is similar to that of the BC95 model, but the association of exogenous determinants with inefficiency is modelled via the variance of u_i. Specifically, the model is formulated as follows

$$TC_i = \alpha + Y_i \beta_Y + Z_i' \beta_Z + \varepsilon_i, \quad i = 1, \dots, n \tag{16}$$

$$\varepsilon_i = v_i + u_i, \tag{17}$$

$$v_i \sim \mathcal{N}(0, \sigma_v^2), \tag{18}$$

$$u_i \sim \mathcal{N}^+\left(0, \sigma_{u_i}^2\right), \tag{19}$$

$$ln\left(\sigma_{u_i}^2\right) = W_i' \, \delta, \tag{20}$$

The limitations of this model are the same as for the BC95 model, except that it appears to suffer less from computational problems compared to the BC95 model because the exogenous determinants on inefficiency are modelled through the second moment rather than the first moment.

4.1.4 Schmidt and Sickles (1984) (hereafter SS84)

This model is cast in the classical panel data fashion, formulated as follows

$$TC_{it} = \alpha + Y_{it} \beta_Y + Z_{it}' \beta_Z + \varepsilon_{it}, \quad i = 1, \cdots, N, \quad t = 1, \cdots T, \tag{21}$$

$$\varepsilon_{it} = v_{it} + u_i, \qquad u_i \geq 0, \tag{22}$$

Thus, the model can be rewritten as

$$TC_{it} = \alpha + Y_{it} \beta_Y + Z_{it}' \beta_Z + v_{it} + u_i = (\alpha + u_i) + Y_{it} \beta_Y + Z_{it}' \beta_Z + v_{it} = \alpha_i + Y_{it} \beta_Y + Z_{it}' \beta_Z + v_{it} \, . \tag{23}$$

After utilising fixed effects to estimate the model and obtain the estimate $\hat{\alpha}_i$ of α_i, the inefficiency can be estimated as follows

$$\hat{u}_i = \hat{\alpha}_i - \min_i \hat{\alpha}_i \, . \tag{24}$$

The SS84 model discussed here is classic and a generalisation of our fixed effects regression model discussed in the preliminary regression. It is natural candidate to be selected as a leading model in our further analysis, where fixed effects outperformed others.

The main advantage of this model is that it does not require distributional assumptions on the error components, unlike most other stochastic frontier models.

A limitation of the model is that inefficiency is assumed to be time-invariant and it does not allow for the inefficiency and other time-invariant individual heterogeneity to be separated. That is, the estimates of inefficiency would effectively absorb all other (observed or unobserved) individual heterogeneity that is fixed over time, some of which may have no relationship to inefficiency but rather being some exogenous characteristics of the individuals (facilities), such as location.

The formulation of inefficiency for the SS84 model implicitly assumes that the most efficient unit in the sample is 100 percent efficient, thus the model is particularly sensitive to outliers/extreme values, such as "super-efficient" observations. In such cases, this approach yields estimates of very high inefficiency (i.e. very low efficiency) and so should be interpreted with caution.

This approach might be too restrictive for a relatively large data set with fairly large individual heterogeneity as in this project. It would, however, be still useful for comparisons and validation of other models with respect to estimates of other key parameters, such as coefficients (or elasticities) of the cost frontier.

4.1.5 Pitt and Lee (1981) (hereafter PL81)

The model is formulated as follows:

$$TC_{it} = \alpha + Y_{it}\beta_Y + Z'_{it}\beta_Z + \varepsilon_{it}, \quad i = 1, \cdots, N, \quad t = 1, \cdots T, \tag{25}$$

$$\varepsilon_{it} = v_{it} + u_i, \tag{26}$$

$$v_{it} \sim \mathcal{N}(0, \sigma_v^2) \tag{27}$$

$$u_i \sim \mathcal{N}^+(0, \sigma_u^2) \tag{28}$$

Compared to the SS84 model, the PL81 model is more restrictive as it imposes distributional assumptions on v_{it} and u_i (thus the model is estimated by maximum likelihood estimation rather than least squares, and so is also heavier computationally).

The PL81 model discussed here is a generalisation of our random effects regression model discussed in the preliminary regression analysis. Similar to the SS84 model, the limitation of the PL81 model is that it does not allow for the inefficiency and time-invariant individual heterogeneity to be separated and that the inefficiency is assumed to be time invariant. An advantage of this model is that it is expected to be less sensitive to outliers/extreme values compared to the SS84 model.

4.1.6 Time-varying inefficiency models with deterministic and stochastic components

The general formulation of the models is as follows

$$TC_{it} = \alpha_i + Y_{it}\beta_Y + Z'_{it}\beta_Z + \varepsilon_{it}, \quad i = 1, \cdots, N, \quad t = 1, \cdots T, \tag{29}$$

$$\varepsilon_{it} = v_{it} + u_{it}, \tag{30}$$

$$u_{it} = G(t)u_i, \tag{31}$$

$$v_{it} \sim \mathcal{N}(0, \sigma_v^2), \tag{32}$$

$$u_i \sim \mathcal{N}^+(0, \sigma_u^2), \tag{33}$$

Different assumptions on the functional form of the deterministic component, $G(t)$, have been proposed in the literature. For example, Kumbhakar (1990) (hereafter K90) assumes

$$G(t) = [1 + exp(\gamma_1 t + \gamma_2 t^2)], \tag{34}$$

Battese and Coelli (1992) (hereafter BC92) assumes

$$G(t) = exp[\gamma(t - T)], \tag{35}$$

and Kumbhakar and Wang (2005) (hereafter KW05) assumes

$$G(t) = exp\big[\gamma\big(t - \underline{t}\big)\big], \tag{36}$$

where $\underline{t}$ is the beginning period of sample.

The main advantage of these models is that they allow inefficiency to be time-varying. The limitation of this approach is that the time-varying component, $G(t)$, is non-stochastic and common for all individuals, besides the specific parametric assumptions imposed on the distributions and the functional relationships.

4.1.7 "True fixed effect" model

This is a more recent and more advanced model that was developed by Greene (2005) and others to mitigate the limitations of the earlier panel data models described above and so it is expected that this model is among the most suitable for this project.

The model is formulated as follows

$$TC_{it} = \alpha_i + Y_{it}\beta_Y + Z_{it}'\beta_Z + \varepsilon_{it}, \quad i = 1,\cdots,N, \quad t = 1,\cdots T, \tag{37}$$

$$\varepsilon_{it} = v_{it} + u_{it}, \tag{38}$$

$$v_{it} \sim \mathcal{N}(0,\sigma_v^2), \tag{39}$$

$$u_{it} \sim \mathcal{N}^+(0,\sigma_u^2). \tag{40}$$

The "True Fixed Effect" model can be viewed as an improvement upon the classic SS84 model. The main advantage of this model is that inefficiency is distinguished from individual heterogeneity. Moreover, the inefficiency is no longer assumed to be fixed over time and is not as sensitive to outliers.

The estimation of this model, however, is not easy. As a result, in this study, two different approaches are applied to estimate the "True Fixed Effect" model as discussed in the following sub-sections.

4.1.7.1 Greene (2005) (hereafter G05)

One approach to estimate the true fixed effects model is to use maximum-likelihood dummy variables proposed by Greene (2005). This approach may be too complicated or infeasible for large data sets with many explanatory variables. G05 is also subject to the incidental parameters problem and often suffers from computational issues since the number of parameters to be estimated can be prohibitively large for a large *N* (as in this project).

4.1.7.2 Chen, Schmidt, and Wang (2014) (hereafter CSW14)

Another approach is to utilise the within maximum-likelihood estimator proposed recently by Chen, Schmidt, and Wang (2014) or CSW14. The CSW14 overcomes the issue of the G05 model by deriving the log-likelihood function of the within transformation, yet the estimation issue is still fairly complex for large samples in relatively large dimensions as in this project.

As for most other stochastic frontier models, a limitation of the "True Fixed Effect" model (whether via G05 or CSW14) is that the specific parametric assumptions imposed on the distributions and the functional relationship for the frontier.

4.1.7.3 Wang and Ho (2010) (hereafter WH10)

The model is formulated as follows

$$C_{it} = \alpha_i + Y_{it}\beta_Y + Z_{it}'\beta_Z + \varepsilon_{it}, \quad i = 1,\cdots,N, \quad t = 1,\cdots T, \tag{41}$$

$$\varepsilon_{it} = v_{it} + u_{it}, \tag{42}$$

$$v_{it} \sim \mathcal{N}(0,\sigma_v^2), \tag{43}$$

$$u_{it} = h_{it}u_i^*, \tag{44}$$

$$h_{it} = W_{it}'\delta, \tag{45}$$

$$u_i^* \sim \mathcal{N}^+(\mu\,,\sigma_u^2), \tag{46}$$

where W_{it} is a vector of exogenous determinant of the inefficiency.

WH10 is another approach to solve the problem of G05. The main advantage of this model is that because α_i is dropped from the model by the within transformation,[18] the computational issue and incidental parameters problem of G05 can be substantially reduced.

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[18] With the specification of the model, the within transformation can be estimated using the standard practice in the literature.

The limitation of this model is that the identification of the model depends largely on the assumption regarding the scaling property of the inefficiency component, besides the specific parametric assumptions imposed on the distributions and the functional relationships for the frontier and the inefficiency.

4.1.8 Kumbhakar, Lien, and Hardaker (2014) (hereafter KLH14)

The model is formulated as follows

$$TC_{it} = \alpha + Y_{it}\beta_Y + Z'_{it}\beta_Z + \mu_i + v_{it} + \eta_i + u_{it}, \quad i = 1, \cdots, N, \quad t = 1, \cdots T, \tag{47}$$

$$\mu_i \sim \mathcal{N}\left(0, \sigma_\mu^2\right) \tag{48}$$

$$\eta_i \sim \mathcal{N}^+\left(0\,, \sigma_\eta^2\right) \tag{49}$$

$$v_{it} \sim \mathcal{N}(0, \sigma_v^2) \tag{50}$$

$$u_{it} \sim \mathcal{N}^+(0\,, \sigma_u^2) \tag{51}$$

where $\eta_i \geq 0$ and $u_{it} \geq 0$ are persistent and time-varying efficiency, and μ_i and v_{it} are firm effects and noise, respectively.

The main advantage of this model is that it allows for decomposing inefficiency into persistent and time-varying components and distinguishing persistent inefficiency from individual heterogeneity. Meanwhile, the limitation of this model is that the identification depends largely on the distributional assumptions on the four error components, besides the specific parametric assumptions imposed on the functional relationships for the cost frontier with respect to other variables.

4.2 Model specifications

In the following analysis, for all stochastic frontier models, a so-called *grand cost frontier* is estimated for all residential aged care facilities in the sample regardless of their characteristics (i.e. only output and year dummies are included in the cost frontier function). The use of a *grand cost frontier* can be justified by the objective to measure the (in)efficiency of *all* residential aged care facilities with respect to a common benchmark, regardless of their characteristics. Stochastic frontier models where characteristics of residential aged care facilities (such as their type) are included in the cost frontier function were also tried. The results are similar and can be found in *Appendix C* of this *Technical Supplement Report 2*.[19]

The characteristics of residential aged care facilities, which are thought to influence the cost structure of residential aged care facilities, were included in the stochastic frontier models that allow for modelling the inefficiency on exogenous explanatory variables (i.e. Models BC95, CFG95, and WH10). In particular, we explored the association of the following characteristics with the inefficiency: occupancy (*occupancy rate)*, remoteness (*remote*), size (*small*), type (*for-profit* and *government*), and quality (*Q1 and Q2*).[20]

4.3 Empirical results

The results are reported for both linear and logarithmic functional forms and for both the raw output and casemix adjusted output (see more details in Section 3.1).[21]

As discussed in Section 4.1, for the linear functional form, the inefficiency, $E(u_i|\, \varepsilon_i)$, is measured in absolute term (e.g. dollars), and thus we also consider the ratio of inefficiency to total cost as a measure of relative inefficiency. Meanwhile, for the logarithmic specification, a convenient measure is relative efficiency, $E[exp(-u_i)|\varepsilon_i]$, estimated using the approach discussed in Battese and Coelli (1988).

The relative inefficiency with the logarithmic specification can be inferred by utilising the following approximation

[19] It is also worth noting that these models were computationally less feasible in the sense of numerical convergence. Intuitively, the reason for lower feasibility was that the estimation procedure required optimising relatively complex log-likelihood functions, and the computation experienced more difficulties if there were too many parameters to be optimised over (as is the case with this dataset, which contains a relatively large number of variables and observations to handle for some maximum likelihood estimations even with modern hardware and software).

[20] Q1 represents RACFs in the highest quality group, and Q2 represents RACFs in the medium high and medium quality groups. We also tried models where quality was included as a variable determining the cost frontier, expecting it to influencing the cost function positively (i.e., higher quality to be associated with higher minimal cost), yet all feasible models could not support this hypothesis (possibly due to quality of the data on quality)

[21] The software used to estimate SFMs is Stata 16. All the models except for CSW14 are estimated using procedures discussed in Kumbhakar, Wang, and Horncastle (2015). CSW14 is estimated using sftfe function provided in Belotti and Ilardi (2018).

$$Relative\ inefficiency \approx 1 - Relative\ efficiency.$$

For all the cases, all the models discussed in Section 4.1 were applied to our data. However, the optimisation algorithms in the available software for some models did not converge after many hours of running and thus were terminated. As a result, while all the models listed above were tested, the results are reported only for model that were feasible in terms of estimation (i.e. if a model is not reported, it means that the maximum likelihood optimisation process for the model did not converge in our estimations).

It is worth noting that due to the current limitations of the estimation procedures available in official software packages, the reported standard errors are not the “robust” or “cluster robust” procedures. They are based on standard procedures in statistics (e.g. Fisher information matrix when maximum likelihood estimation is used) and are theoretically justified under certain assumptions of the data generating process for the corresponding models. This is described in more detail in the respective journal article cited for each approach.

As a result, the standard errors here are smaller than those that are reported in the analysis of the average cost function. If requested, approximate corrections for the robustness and clustering of the standard errors can be developed by adapting ideas from the most current non- stochastic frontier analysis literature on clustering cited above.

For ease of comparisons among estimated models, facilities which only have data in a single year were excluded, which results in a small reduction in the sample size (169 out of 5,880 observations were excluded).[22]

4.3.1 Linear form

4.3.1.1 Raw output (the total number of bed days)

As shown in Table 10, five models are feasible for linear function form with raw output. Among the five models, the results of the ALS77 and CFG95 models are very similar in terms of estimated coefficients of the frontier as well as estimated inefficiency. The inefficiency for an average facility estimated by the two models are around $1.1M per year (i.e. around 20%) (Table 11). Similarly, the PL81 and BC92 models also have similar estimated results. As shown in Table 11, the average inefficiency levels estimated by the PL81 and BC92 models are around 25%.

The SS84 model results in a quite high level of inefficiency. This might be attributed to the fact that a caveat of the SS84 model is that the individual inefficiency scores in this model are assumed to be fixed over time and also absorbs the effects of any other time-invariant variables. For this reason, while the SS84 model is an appealing approach, due to fairly wide heterogeneity that we observe in the sample in our preliminary regression analysis, this model may be largely overestimating the inefficiency in this industry. It is however still useful as a step towards (and comparison to) the so-called “true fixed effect” model.

Regarding the association of residential aged care facility characteristics with inefficiency, it is worth noting that the maximum likelihood estimates reported in Table 10 are not the marginal effects of explanatory variables on the mean inefficiency as the relationships between the explanatory variables and the mean inefficiency are nonlinear. As a result, the average marginal effects are calculated separately and reported in Table 12.[23]

From the estimated results (Table 10), we can see that the coefficients of variables *occupancy rate*, *government*, *small*, *Q1 and Q2* are statistically significant. Therefore, only the marginal effects of these variables are discussed. Specifically, the estimates from the model suggest that the mean inefficiency is expected to decrease by $10,678, on average and ceteris paribus, if the occupancy rate increases by 1%. The model also suggests that the mean inefficiency of government facilities is $408,678 larger than that of not-for-profit facilities, on average and ceteris paribus. Meanwhile, the mean inefficiency of small facilities is, on average and ceteris paribus, $697,878 lower than the mean inefficiency of non-small facilities.[24]

Interestingly, the estimates from the model also suggests that, on average and ceteris paribus, the mean inefficiencies of Q2 and Q1 facilities are $204,460 and $310,598 less than that of Q3 facilities, respectively. Models, where quality was included as a variable determining the cost frontier, were also tested and were

[22] This is because the estimation procedure for CSW14 automatically drops observations which only have data in a single year. We also estimated the other models with the full sample to compare results and found that the results are very similar and conclusions are robust across the samples we used.

[23] More details about the calculation of the average marginal effects can be found in Kumbhakar, Wang, and Horncastle (2015).

[24] A facility is classified as *small* if it has less than 30 places.

expected to influence the cost function positively (i.e. higher quality to be associated with higher minimal cost). However, all feasible models could not support this hypothesis (possibly due to quality of the data on the quality of residential aged care facilities).

Table 10. Frontier estimation for linear functional form (raw output)

Total cost	**ALS77**	**CFG95**	**PL81**	**BC92**	**SS84**
Frontier					
Y (occupied bed days)	245.19*** (1.155)	243.23*** (1.23)	217.11*** (1.826)	217.4*** (1.853)	204.06*** (2.977)
Financial year 2014/15	-546889.4*** (47774.21)	-557362.7*** (46494.87)	-485054.2*** (23427.38)	-510165.8*** (37361.05)	-495235*** (22833.63)
Financial year 2015/16	-413213.2*** (47141.8)	-424345.5*** (45895.35)	-366994.6*** (23111.47)	-385855.4*** (31780.68)	-372366.2*** (22473.26)
Financial year 2016/17	-288379.4*** (46303.02)	-299004.1*** (45087.14)	-239332.9*** (22442.91)	-251999.6*** (26784.04)	-243715.7*** (21771.16)
Financial year 2017/18	-149061.9*** (46141.44)	-166716.5*** (44985.17)	-129036*** (22234.34)	-135586.8*** (23477.34)	-134385.9*** (21556.72)
Constant	-616434.9*** (45596.36)	-533546.7*** (45489.54)	-370782.1*** (49717.68)	-363847.4*** (50313.04)	1607702*** (82132.04)
Log Likelihood	-87,595	-87,477	-85,071	-85,071	-81,991
$\ln(\sigma_u^2)$ ***(Inefficiency)***					
Constant	28.44*** (0.038)	30.42*** (0.24)	29.06*** (0.047)	29.04*** (0.05)	
Occupancy rate		-0.02*** (0.002)			
For-profit		-0.03 (0.07)			
Government		0.7*** (0.11)			
Remote		0.35 (0.22)			
Small		-1.19*** (0.14)			
Q1		-0.53*** (0.1)			
Q2		-0.35*** (0.07)			
$ln(\sigma_v^2)$ ***(Residuals)***					
Constant	26.9*** (0.046)	26.86*** (0.05)	26.23*** (0.022)	26.23*** (0.022)	
G(t)					
$t - T$				-0.004 (0.005)	

ALS77: Aigner, Lovell and Schmidt (1977); CFG95: Caudill, Ford, and Gropper (1995); PL81: Pitt and Lee (1981); BC92: Battese and Coelli (1992); SS84: Schmidt and Sickles (1984); See Section 4.1.
G(t): time-varying component; t: time, T: the end period of the sample.
The base case is: financial year 2018/19, not-for-profit, not remote, 30+ beds, Q3.
Standard errors in parentheses
* p < 0.10; ** p < 0.05; *** p < 0.01

Table 11. Estimated inefficiency for linear functional form, raw output

	Model	Observations	Mean	Standard deviation	Min	Max	Histogram
Inefficiency	ALS77	5,711	1,158,401	786,351	103,126	8,304,783	Figure 3
	CFG95	5,711	1,138,274	808,740	101,331	8,205,243	Figure 5
	PL81	5,711	1,625,003	1,150,509	26,679	9,569,733	Figure 7
	BC92	5,711	1,623,521	1,148,589	26,354	9,611,443	Figure 9
	SS84	5,711	3,756,797	1,291,747	0	11,900,000	Figure 11
Inefficiency relative to total cost	ALS77	5,711	0.20	0.14	0.01	1.85	Figure 4
	CFG95	5,711	0.19	0.12	0.01	1.27	Figure 6
	PL81	5,711	0.25	0.14	0.00	1.28	Figure 8
	BC92	5,711	0.25	0.14	0.00	1.29	Figure 10
	SS84	5,711	0.67	0.35	0.00	4.72	Figure 12

ALS77: Aigner, Lovell and Schmidt (1977); CFG95: Caudill, Ford, and Gropper (1995); PL81: Pitt and Lee (1981); BC92: Battese and Coelli (1992); SS84: Schmidt and Sickles (1984). See Section 4.1.

Table 12. Marginal effects of explanatory variables on mean inefficiency for linear functional form, raw output

	CFG95 Marginal Effects	Simple interpretation of the marginal effect
Occupancy rate	-10,678	Lower cost inefficiency
For-profit	-16,190	Lower cost inefficiency
Government	408,678	Higher cost inefficiency
Remote	207,281	Higher cost inefficiency
Small	-697,878	Lower cost inefficiency
Q1	-310,598	Lower cost inefficiency
Q2	-204,460	Lower cost inefficiency

CFG95: Caudill, Ford, and Gropper (1995). See Section 4.1.
The base case is: not-for-profit, not remote, 30+ beds, Q3

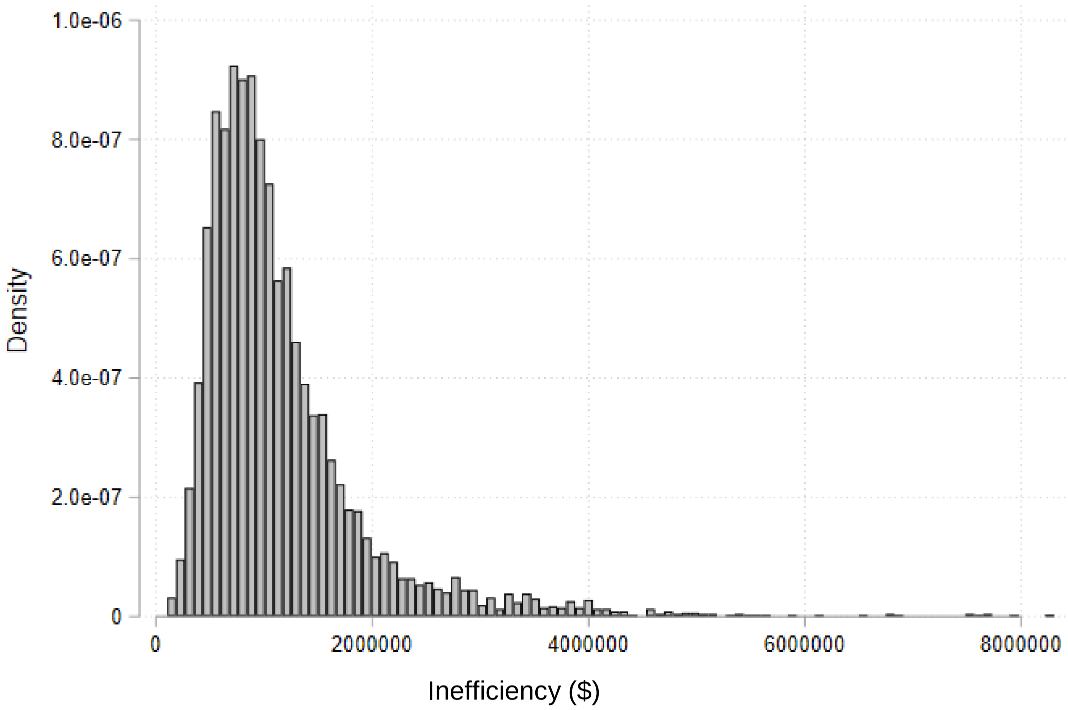

Figure 3. Histogram of estimated inefficiency for ALS77 model, linear functional form, raw output

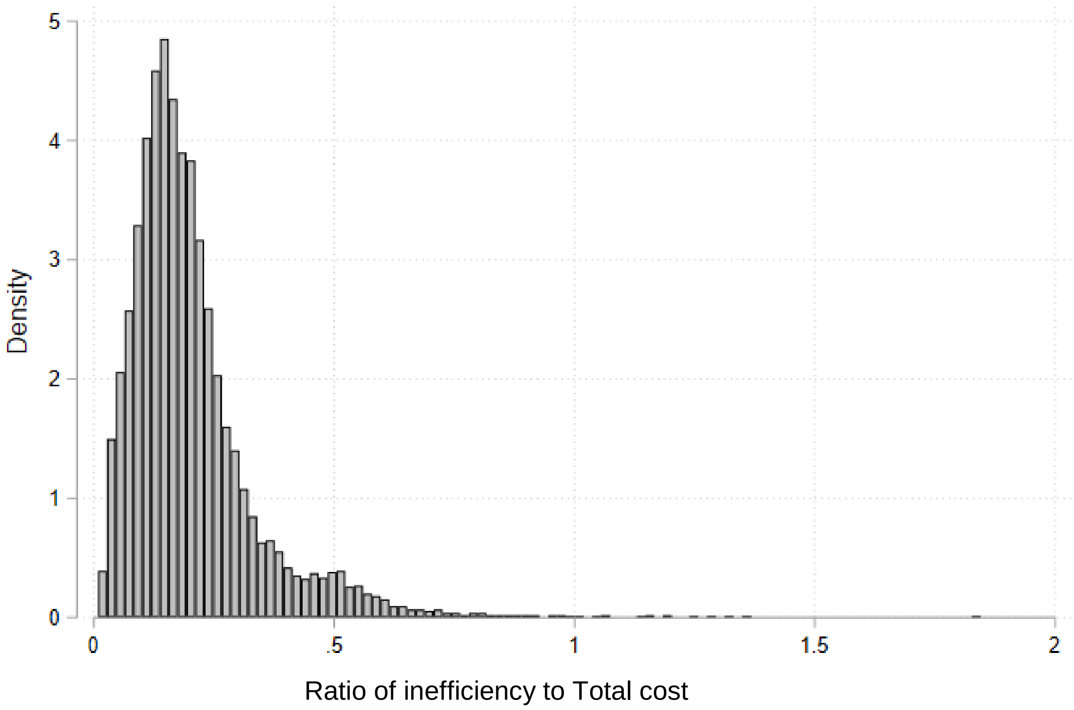

Figure 4. Histogram of ratio of inefficiency to total cost for ALS77 model, linear functional form, raw output

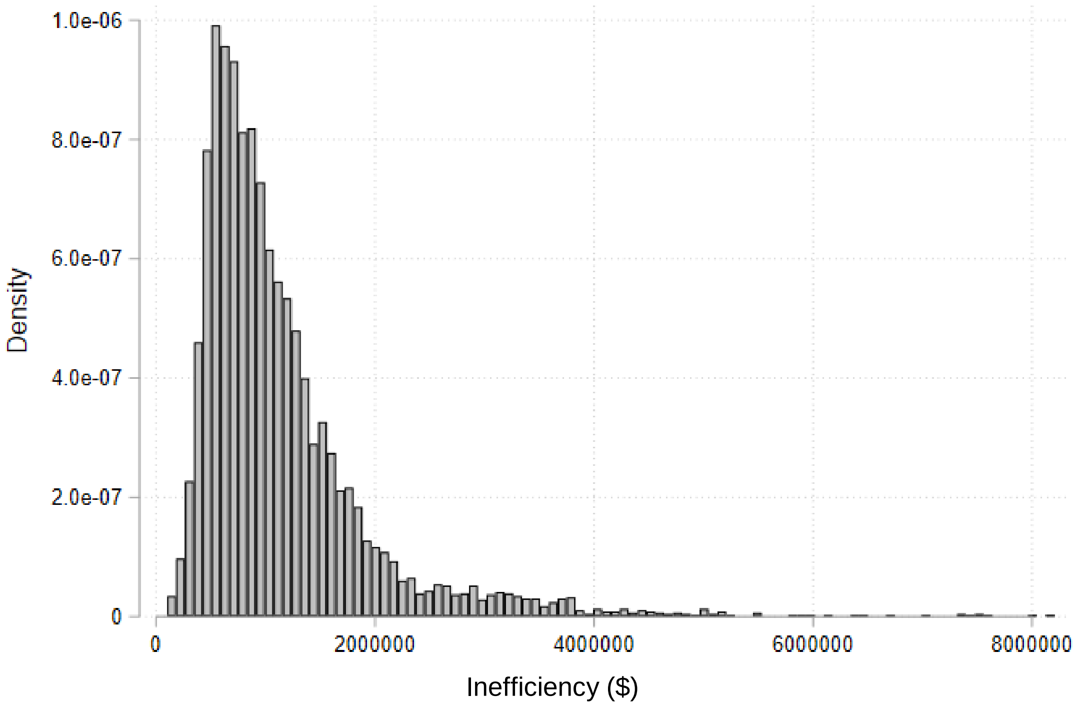

Figure 5. Histogram of estimated inefficiency for CFG95 model, linear functional form, raw output

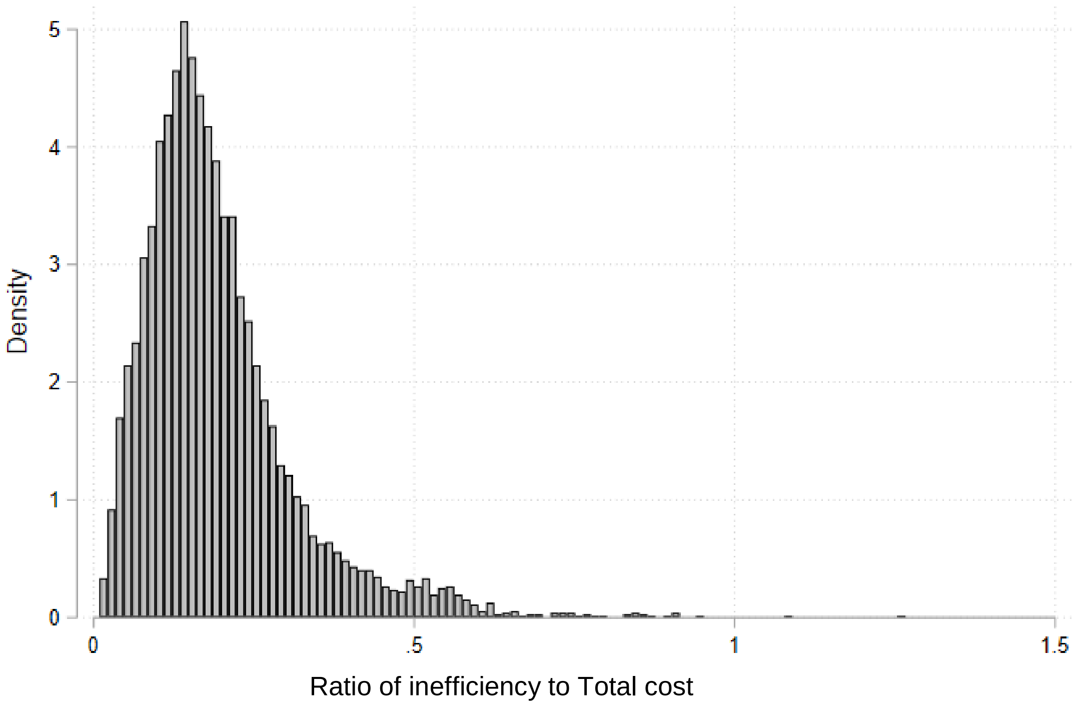

Figure 6. Histogram of ratio of inefficiency to total cost for CFG95 model, linear functional form, raw output

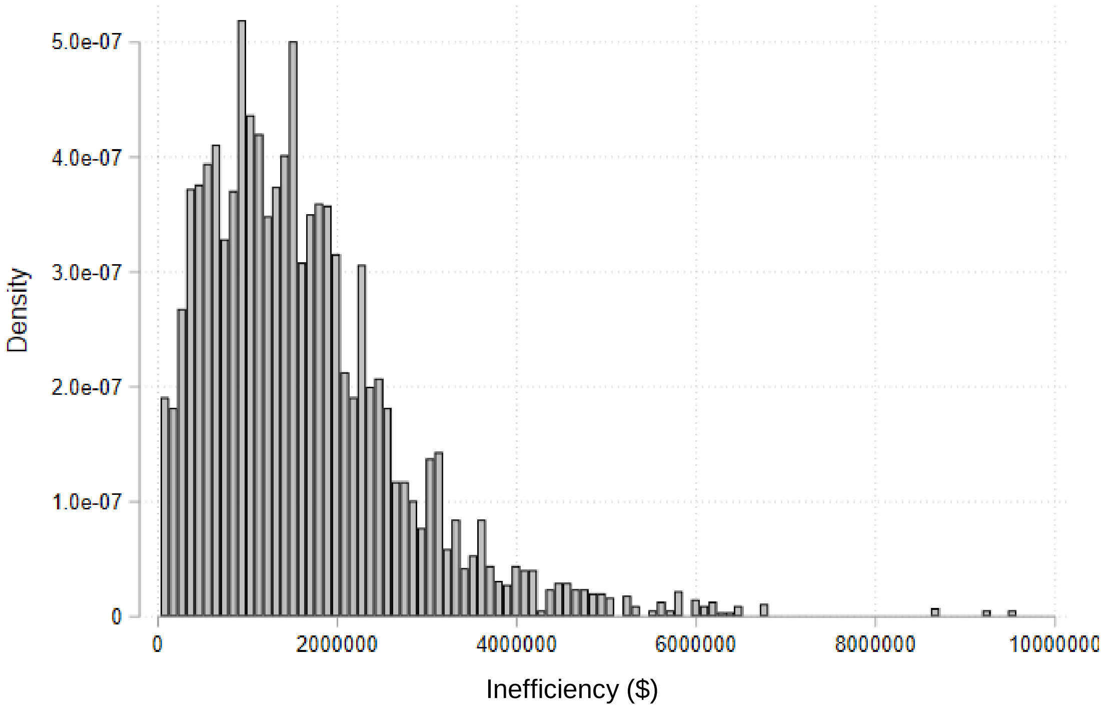

Figure 7. Histogram of estimated inefficiency for PL81 model, linear functional form, raw output

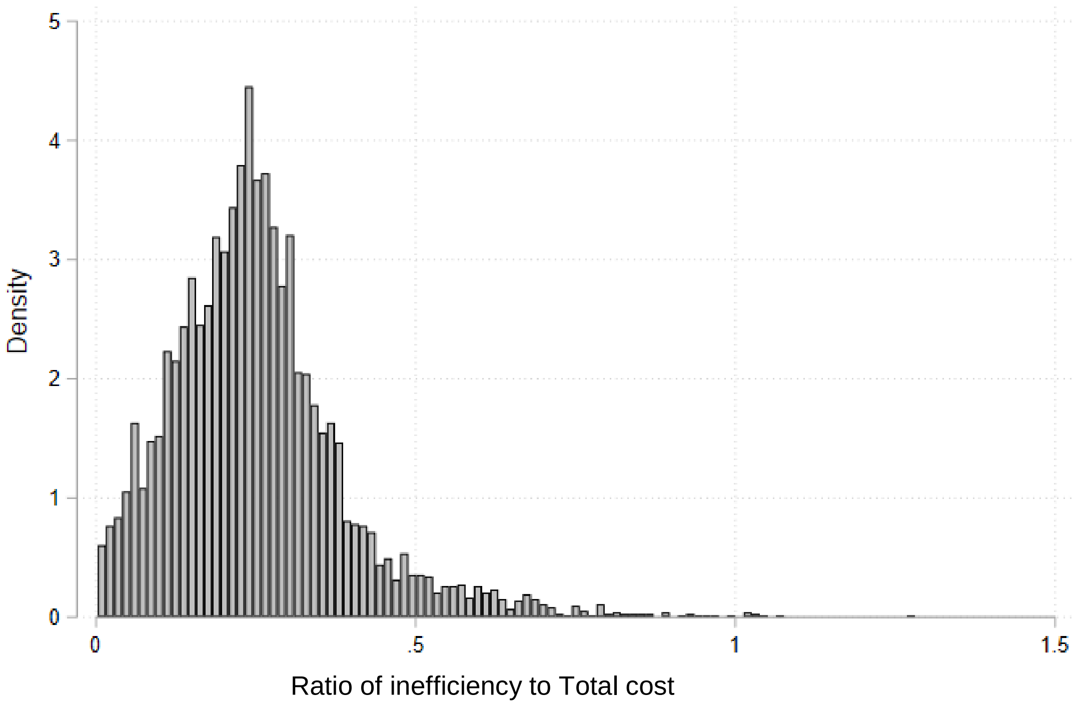

Figure 8. Histogram of ratio of inefficiency to total cost for PL81 model, linear functional form, raw output

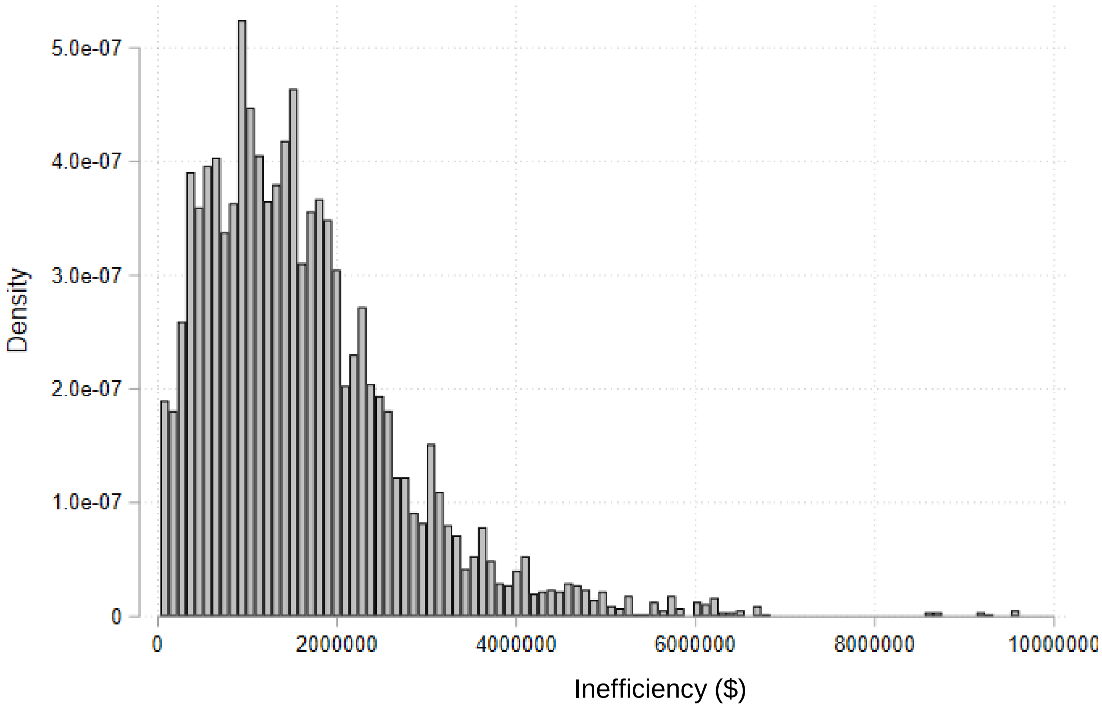

Figure 9. Histogram of estimated inefficiency for BC92 model, linear functional form, raw output

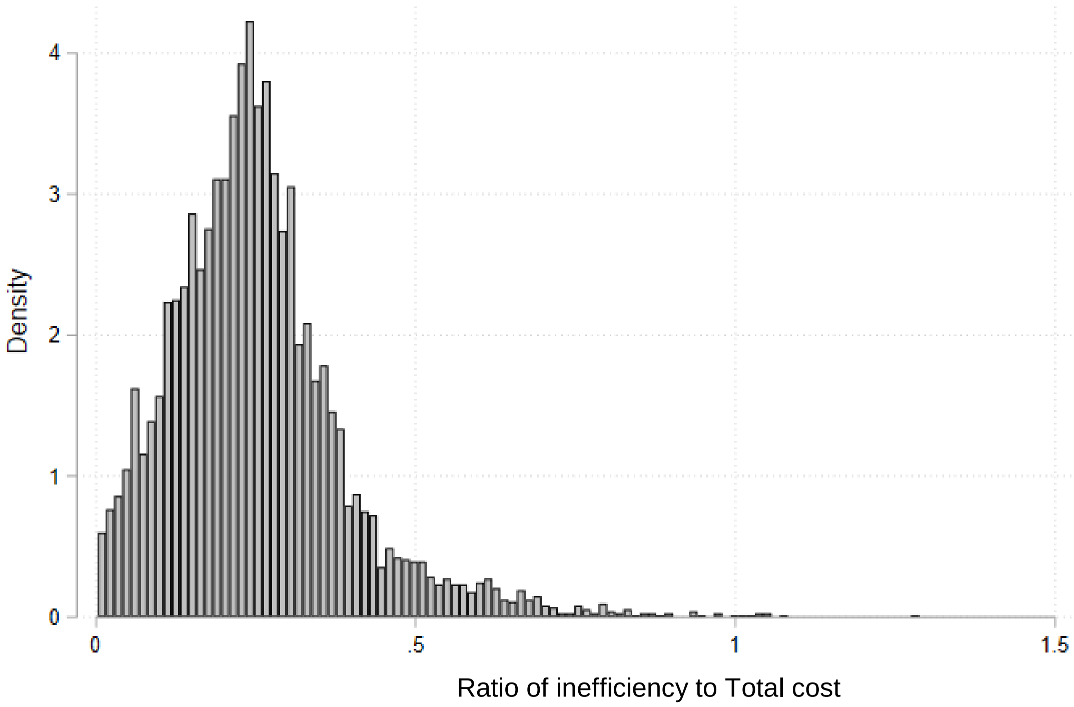

Figure 10. Histogram of ratio of inefficiency to total cost for BC92 model, linear functional form, raw output

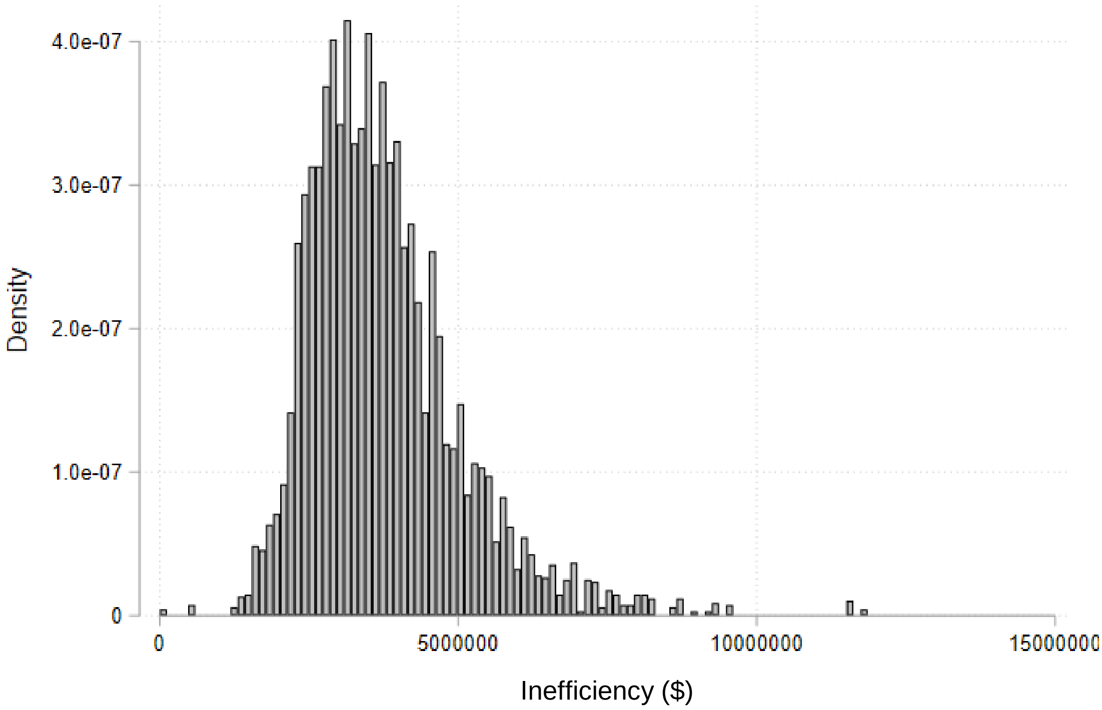

Figure 11. Histogram of estimated inefficiency for SS84 model, linear functional form, raw output

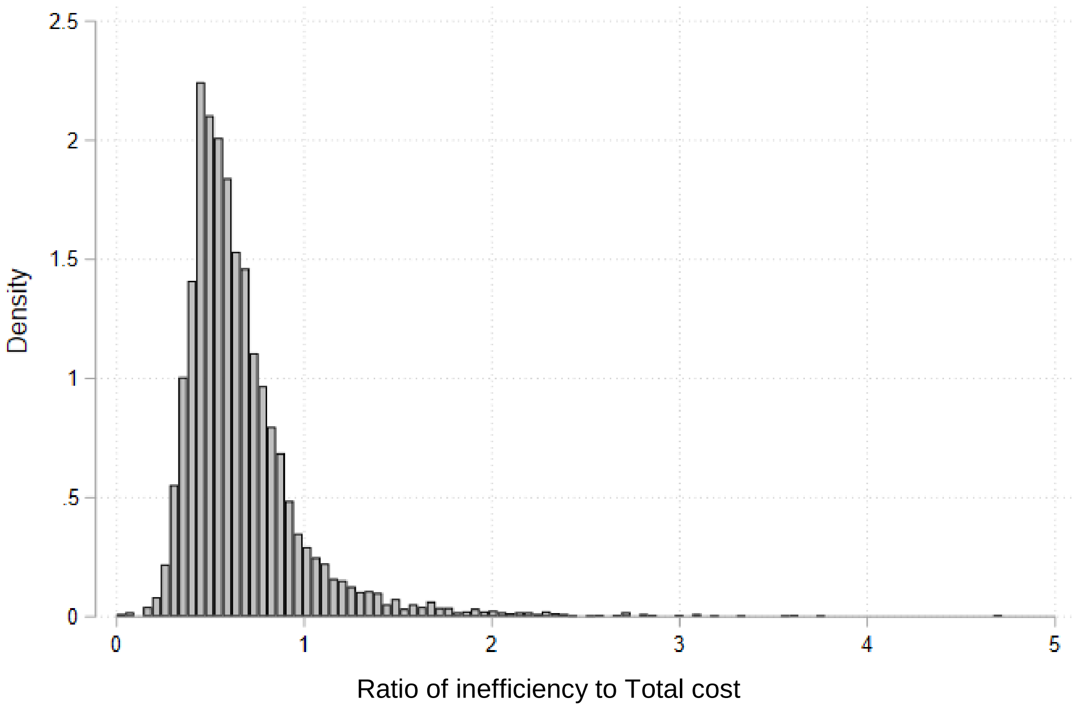

Figure 12. Histogram of ratio of inefficiency to total cost for SS84 model, linear functional form, raw output

4.3.1.2 Casemix adjusted output

As shown in Table 13, five models are feasible for linear function form with casemix adjusted output. However, there seems to be a degeneracy of the computation for the G05 model, where extremely small standard errors of inefficiency are observed (Table 14). Among the four remaining models, the results of the ALS77 and CFG95 models are very similar in terms of estimated coefficients of the frontier as well as estimated inefficiency. The inefficiency for an average facility estimated by the two models are around $1.2M per year (i.e. around 20%).

Similar to the case of raw output, the SS84 model results in a quite a high level of inefficiency and again it can be explained due to the caveat of this model as mentioned in Section 2. Unfortunately, the models that help to address this caveat in panel data setting (e.g. G05, CSWS14, and WH10) did not work for our data. Regarding the association of residential aged care facility characteristics with inefficiency, the estimated average marginal effects of explanatory variables on the mean inefficiency are reported in Table 15.

From the estimated results (Table 13), the coefficients of variables *occupancy rate*, *for-profit*, *government*, *small, Q1 and Q2* are shown to be statistically significant. Therefore, only the marginal effects of these variables are discussion. Specifically, the estimates of the model suggest the mean inefficiency is expected to decrease by $7,662, on average and ceteris paribus, if the occupancy rate increases by 1%. Moreover, the mean inefficiency of government facilities is, on average and ceteris paribus, $199,955 higher than the mean inefficiency of not-for-profit facilities. Meanwhile, the mean inefficiency of for-profit facilities is $82,011 lower than that of not-for-profit facilities, on average and ceteris paribus. Similarly, small facilities have the mean inefficiency being $1,224,393 lower than the mean inefficiency of non-small facilities, on average and ceteris paribus.

We remind the reader that the differences between for-profit, not-for-profit and government should be interpreted with great caution as they may (and perhaps are likely to) reflect differences in quality achieved by the different ownership types which have not been able to be distinguished within the three quality levels by the composite quality index.

Regarding the quality, on average and ceteris paribus, the mean inefficiencies of Q2 and Q1 facilities are $188,895 and $269,922 lower than that of Q3 facilities, respectively. The finding about the significant positive association between efficiency and quality (or negative association between quality and inefficiency) is consistent with the case of raw output discussed in Section 4.3.1.1.

Table 13. Frontier estimation for linear functional form, casemix adjusted output

Total cost	ALS77	CFG95	BC92	SS84	G05
Frontier					
Y* (casemix adjusted occupied bed days)	213.78*** (1.015)	209.17*** (1.05)	198.08*** (1.429)	149.76*** (2.612)	149.76*** (2.267)
Financial year 2014/15	-293649.2*** (38020.65)	-286433.3*** (35614.94)	-165232.4*** (36684.02)	-322825.8*** (25403.65)	-322825.8*** (22050.87)
Financial year 2015/16	-283125.7*** (37506.96)	-270077.1*** (35212.73)	-194140.2*** (32162.34)	-283562*** (24603.62)	-283562*** (21356.42)
Financial year 2016/17	-155506.6*** (37022.67)	-151766.3*** (34889.43)	-75313.67*** (27947.16)	-148347.8*** (23828.02)	-148347.8*** (20683.19)
Financial year 2017/18	-100531.2*** (36685.61)	-100838.4*** (34621.71)	-45823.16* (25175.85)	-87149.95*** (23516.56)	-87149.95*** (20412.84)
Constant	2841.59 (36264.78)	120526*** (34695.07)	339155.9*** (41088.18)	2947484*** (75005.38)	605803.2 (1125241)
Log Likelihood	-86,866	-86,707	-85,224	-82,478	-82,478
$ln(\sigma_u^2)$ (inefficiency)					
Constant	28.579*** (0.029)	30.03*** (0.22)	28.69*** (0.051)		0.856 (1808525)
Occupancy		-0.01*** (0.002)			
For-profit		-0.13** (0.06)			
Government		0.31*** (0.1)			
Remote		0.13 (0.21)			
Small		-1.91*** (0.12)			
Q1		-0.42***			

Total cost	ALS77	CFG95	BC92	SS84	G05
		(0.09)			
Q2		-0.29***			
		(0.06)			
$ln(\sigma_v^2)$ (Residuals)					
Constant	25.77***		26.39***		26.05***
	(0.074)		(0.022)		(0.019)
$G(t)$					
$t - T$			0.014**		
			(0.006)		

ALS77: Aigner, Lovell and Schmidt (1977); CFG95: Caudill, Ford, and Gropper (1995); BC92: Battese and Coelli (1992); SS84: Schmidt and Sickles (1984); G05: Greene (2005). See section 4.1.
G(t): time-varying component; t: time, T: the end period of the sample.
The base case is: financial year 2018/19, not-for-profit, not remote, 30+ beds, Q3.
Standard errors in parentheses
* p < 0.10; ** p < 0.05; *** p < 0.01

Table 14. Estimated inefficiency for linear functional form, casemix adjusted output

	Model	Observations	Mean	Standard deviation	Min	Max	Histogram
Inefficiency	ALS77	5,711	1,224,357	979,304	97,767	8,438,740	Figure 13
	CFG95	5,711	1,225,605	1,023,939	93,980	8,612,661	Figure 15
	BC92	5,711	1,236,260	1,037,420	111,905	8,059,810	Figure 17
	SS84	5,711	2,565,881	1,584,049	0	11,700,000	Figure 19
	G05	5,711	1	0	1	1	Figure 21
Inefficiency relative to total cost	ALS77	5,711	0.20	0.13	0.01	1.03	Figure 14
	CFG95	5,711	0.19	0.11	0.01	0.71	Figure 16
	BC92	5,711	0.18	0.10	0.01	1.02	Figure 18
	SS84	5,711	0.38	0.09	0.00	1.63	Figure 20
	G05	5,711	0.00	0.00	0.00	0.00	Figure 22

ALS77: Aigner, Lovell and Schmidt (1977); CFG95: Caudill, Ford, and Gropper (1995); BC92: Battese and Coelli (1992); SS84: Schmidt and Sickles (1984); G05: Greene (2005). See Section 4.1.

Table 15. Marginal effects of explanatory variables on mean inefficiency for linear functional form, casemix adjusted output

	CFG95 Marginal Effects	Simple interpretation of the marginal effect
Occupancy rate	-7,662	Lower cost inefficiency
For-profit	-82,011	Lower cost inefficiency
Government	199,955	Higher cost inefficiency
Remote	84,677	Higher cost inefficiency
Small	-1,224,393	Lower cost inefficiency
Q1	-269,922	Lower cost inefficiency
Q2	-188,895	Lower cost inefficiency

CFG95: Caudill, Ford, and Gropper (1995). See Section 4.1.
The base case is: not-for-profit, not remote, 30+ beds, Q3

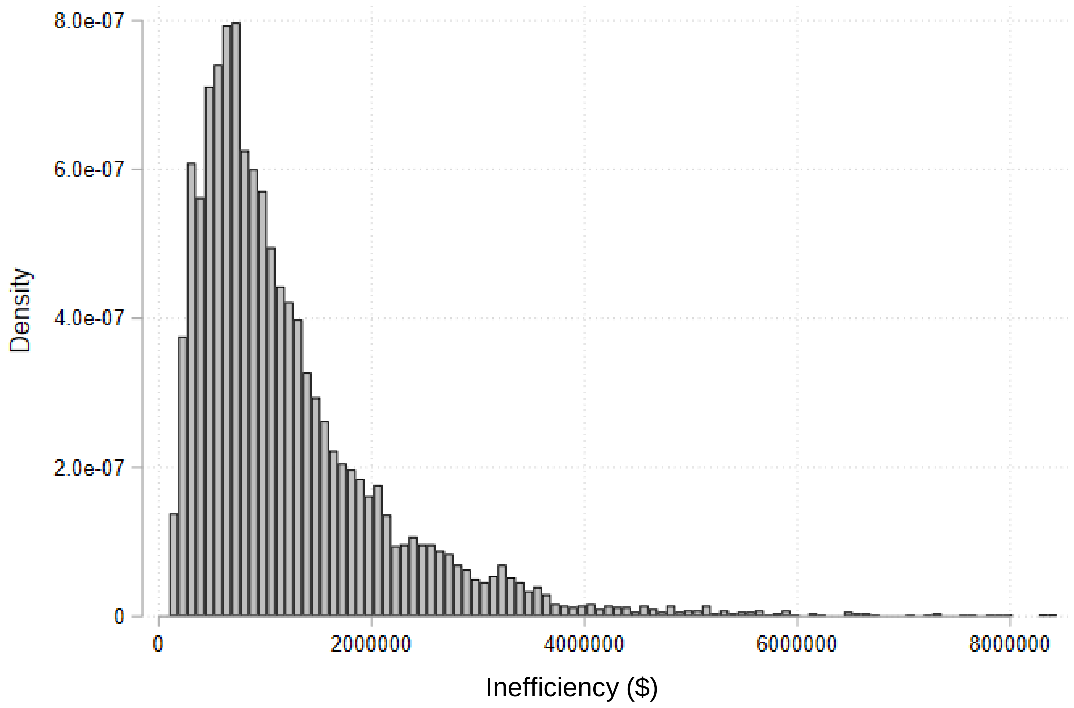

Figure 13. Histogram of estimated inefficiency for ALS77 model, linear functional form, casemix adjusted output

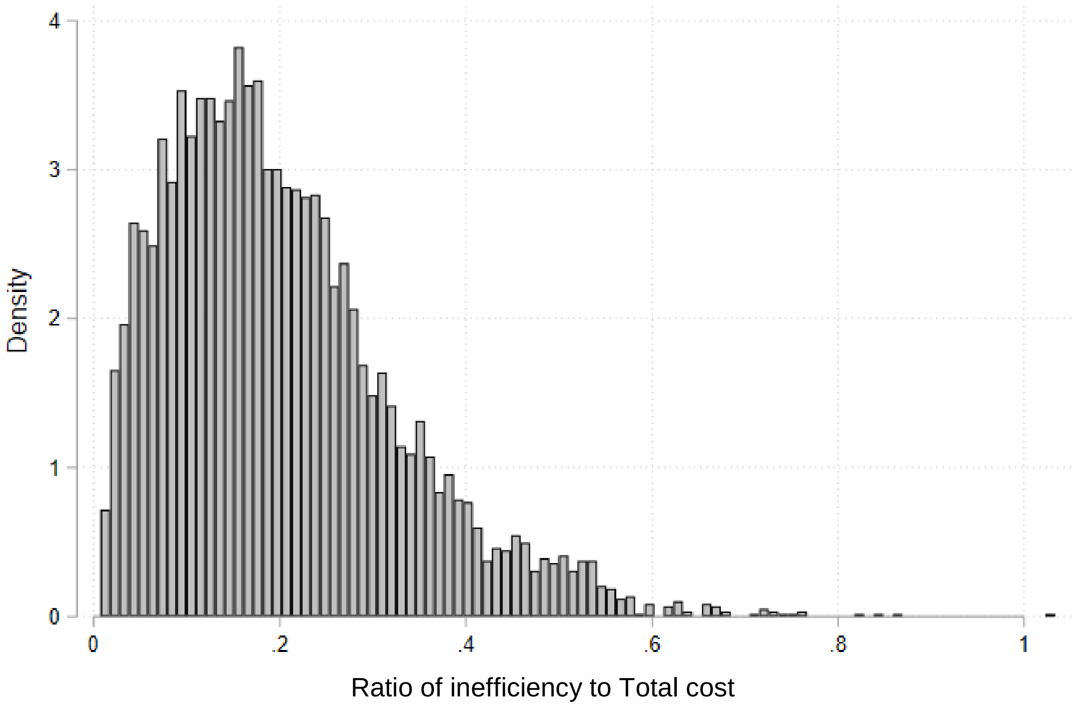

Figure 14. Histogram of ratio of inefficiency to total cost for ALS77 model, linear functional form, casemix adjusted output

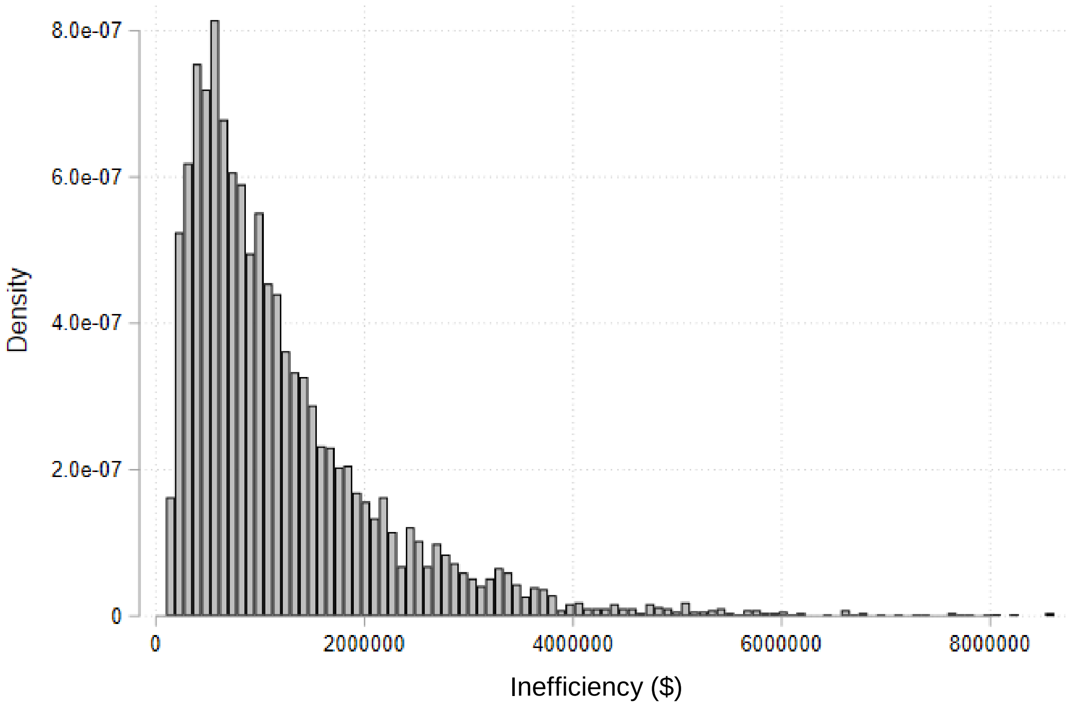

Figure 15. Histogram of estimated inefficiency for CFG95 model, linear functional form, casemix adjusted output

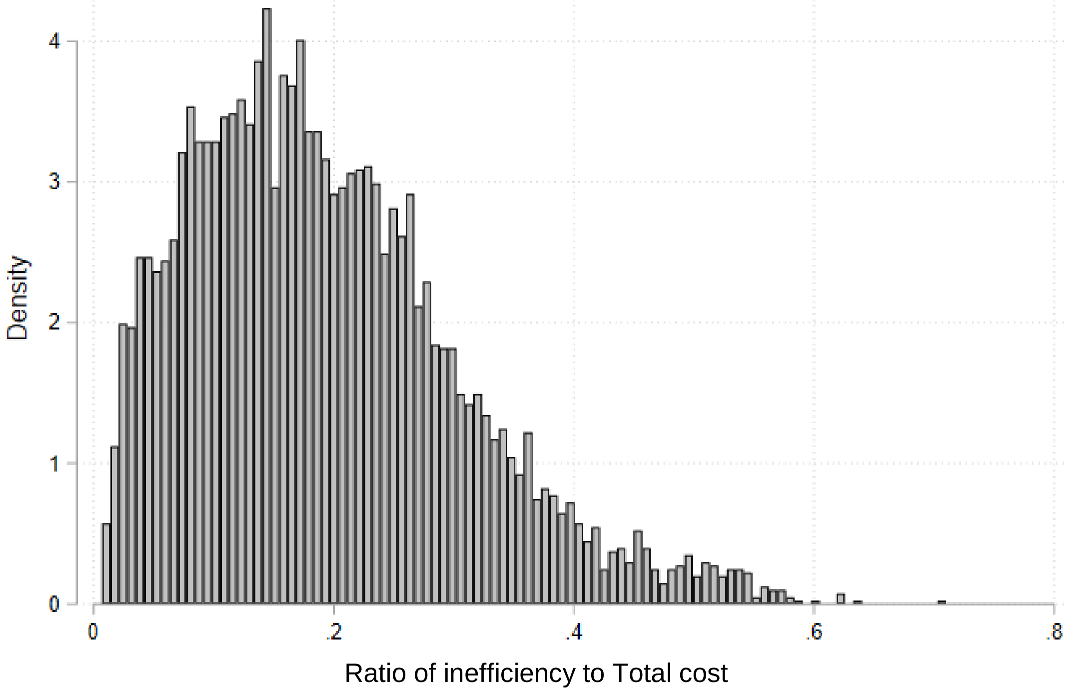

Figure 16. Histogram of ratio of inefficiency to total cost for CFG95 model, linear functional form, casemix adjusted output

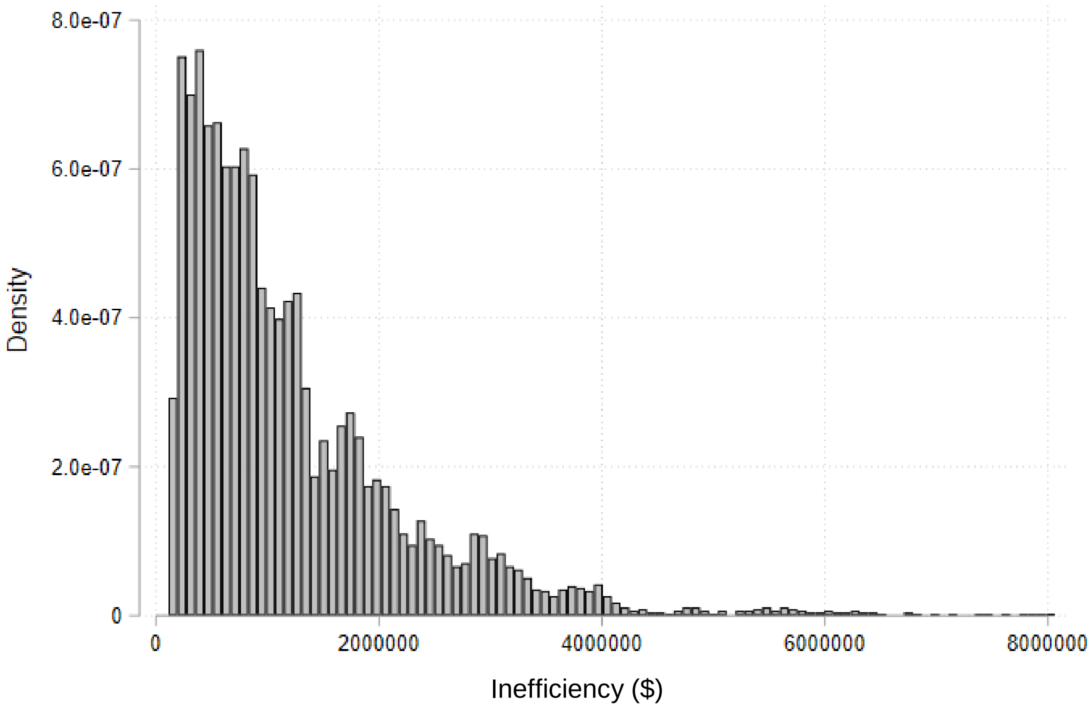

Figure 17. Histogram of estimated inefficiency for BC92 model, linear functional form, casemix adjusted output

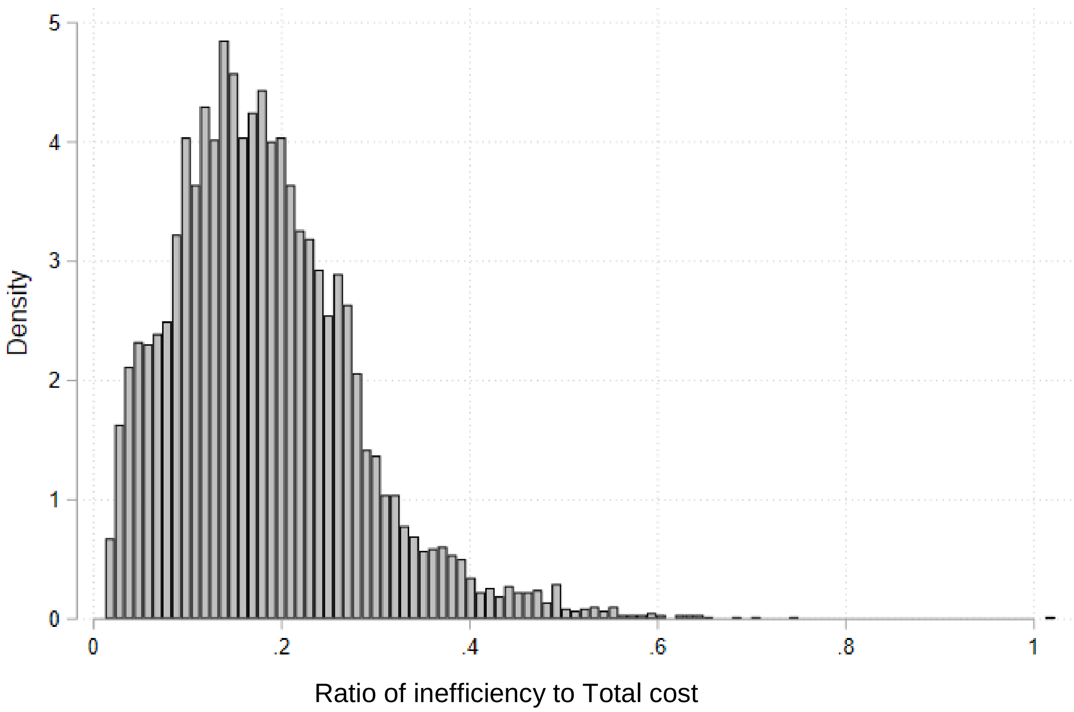

Figure 18. Histogram of ratio of inefficiency to total cost for BC92 model, linear functional form, casemix adjusted output

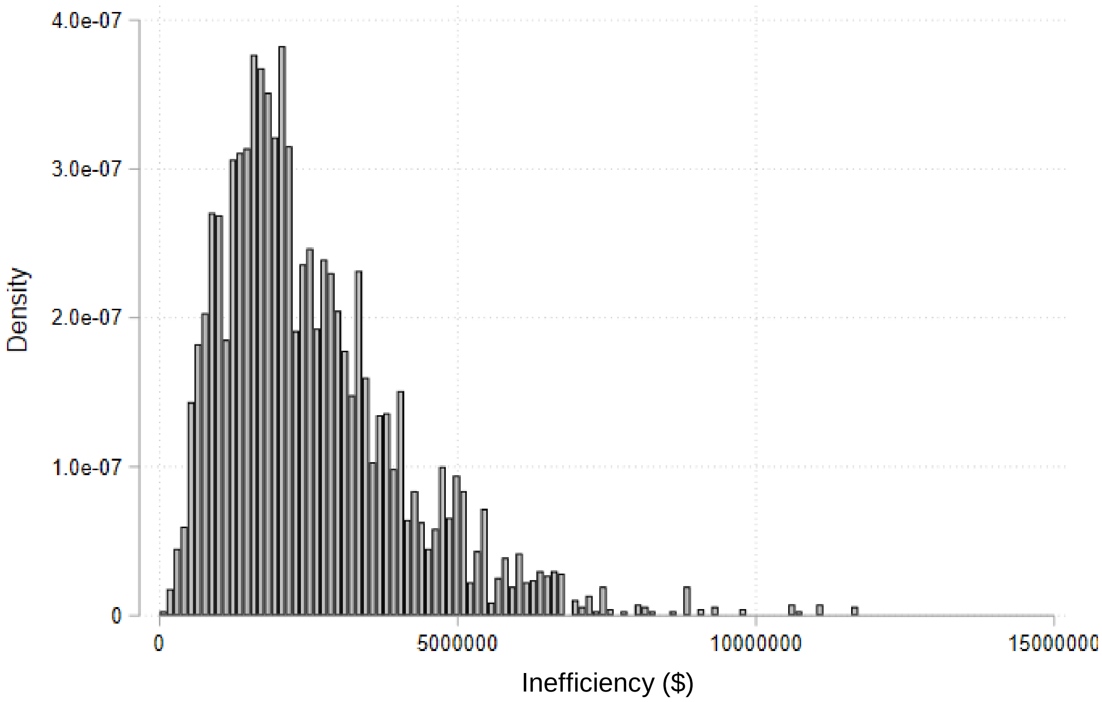

Figure 19. Histogram of estimated inefficiency for SS84 model, linear functional form, casemix adjusted output

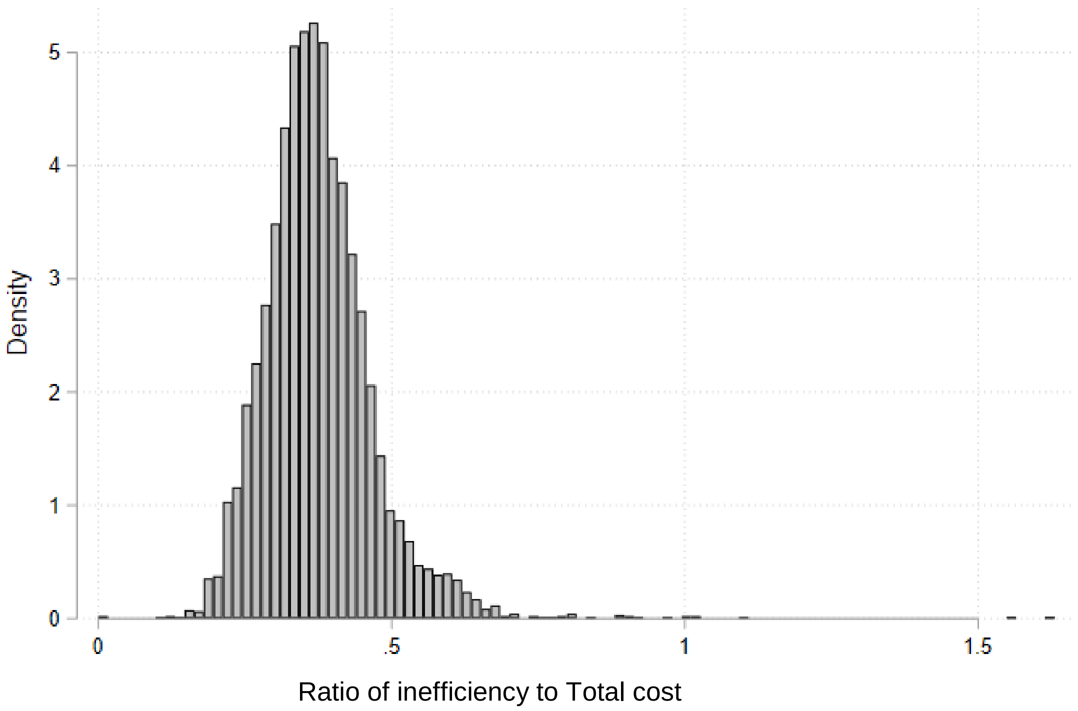

Figure 20. Histogram of ratio of inefficiency to total cost for SS84 model, linear functional form, casemix adjusted output

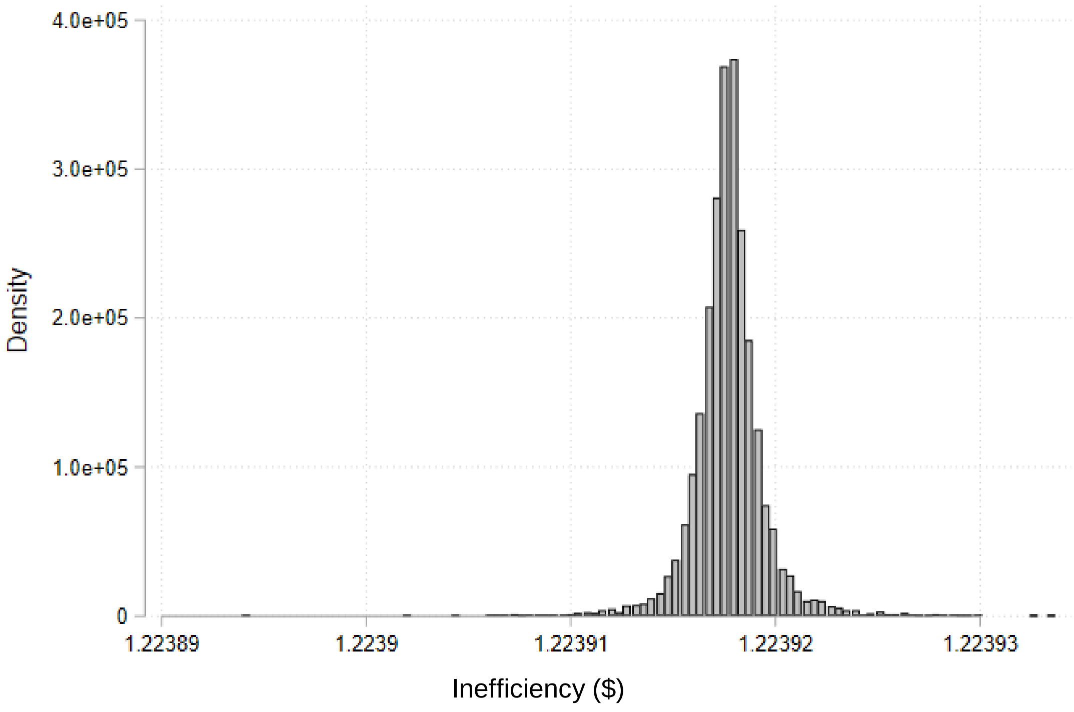

Figure 21. Histogram of estimated inefficiency for G05 model, linear functional form, casemix adjusted output

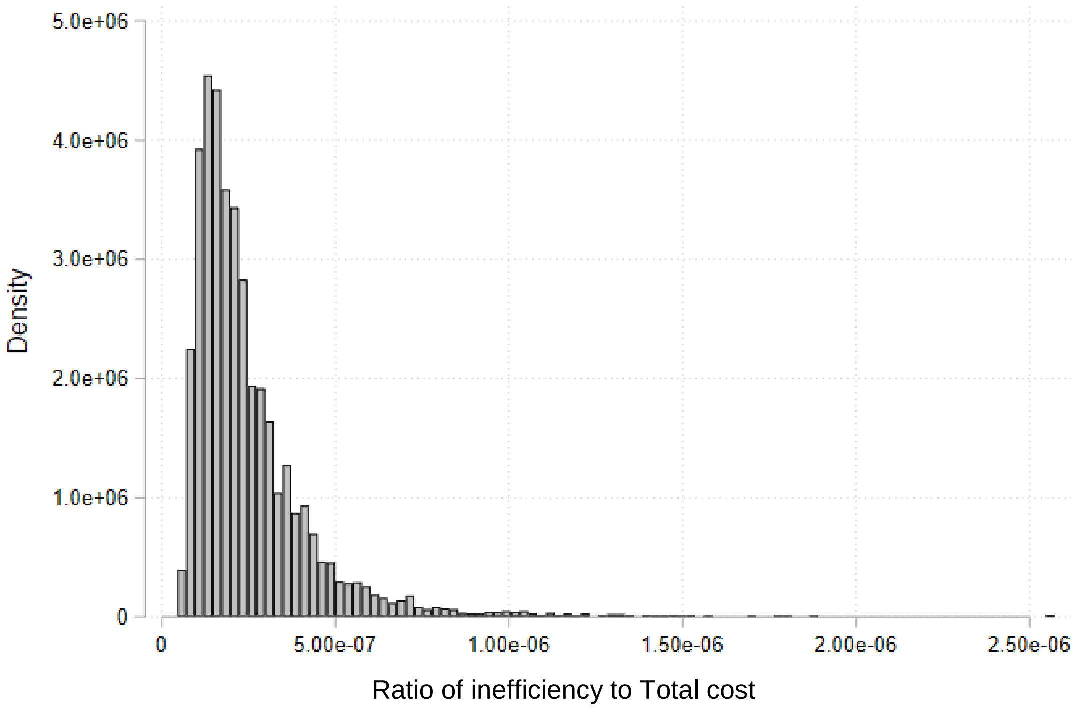

Figure 22. Histogram of ratio of inefficiency to total cost for G05 model, linear functional form, casemix adjusted output

4.3.2 Logarithmic form

4.3.2.1 Raw output (the total number of bed days)

The frontier estimation for logarithmic functional form (raw output) is provided in Table 16.

First, the higher number of feasible models (7 models) suggest that logarithmic functional form might be a more suitable functional form to perform frontier analysis for the data in this project. As discussed in Section 3.5, an advantage of this functional form is that it reduces the impact of potential heteroskedasticity in the data and sparsity at the tails due to shrinking the scale.[25] For this dataset, the log-log specification helps to handle the heteroskedasticity well. The plot of OLS residuals of the linear cost function against output (the fan-shaped pattern on the left panel in Figure 23) shows the evidence of heteroskedasticity of the error terms. The pattern in the residual plot disappears when the log transformation is applied to the data (see the right panel in Figure 23).

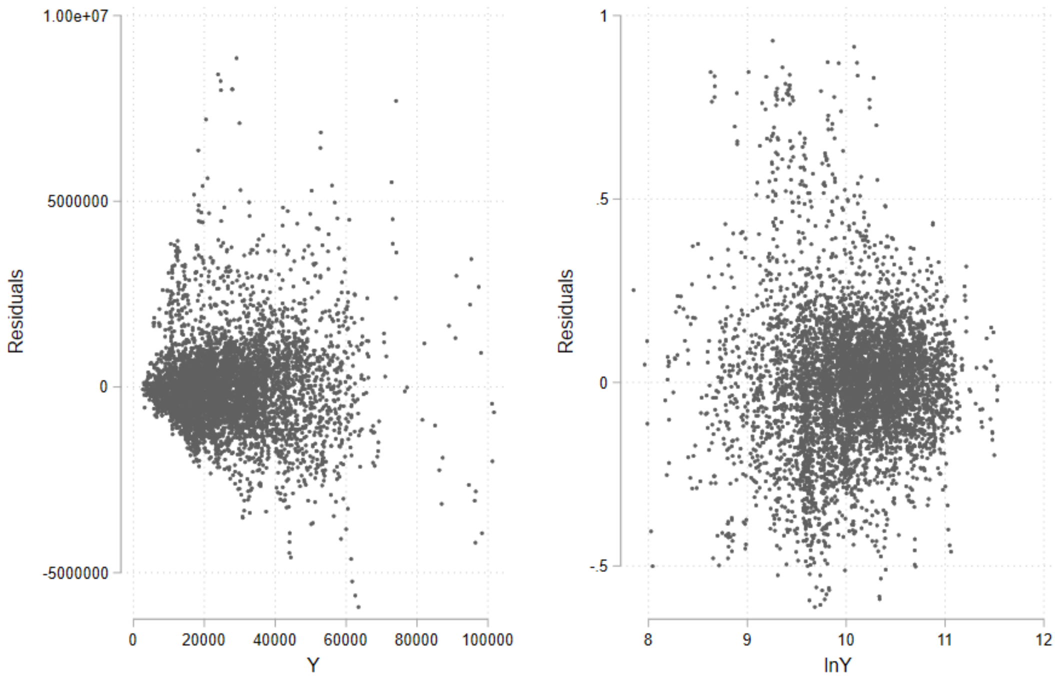

Figure 23. OLS residual plots: linear vs. logarithmic functional form

Second, the estimated coefficients of the frontier (especially coefficients of lnY) are consistent between models that follow the same estimation approach. For example, the ALS77 and CFG95 (cross sectional or pooled data approach), K90 and KW05 (time-varying models), and SS84 and CSW14 (Fixed Effect models) models. As expected, the estimated coefficients of the frontier models are also consistent with the corresponding regression models in the preliminary regression analysis.

Third, a large difference is observed for the elasticity of total cost with respect to output between the pooled data approach and fixed effects approach. The pooled data approaches here are likely to overestimate this coefficient due to not accounting for unobserved heterogeneity over the individuals (i.e. facilities), which to some extent is modelled by the fixed effects approach.

Regarding the estimated efficiency, looking at the histograms, the distributions of estimated efficiency scores are reasonable for all models, except the SS84 model (which might be due to its caveats explained above).

The estimated average efficiency according to the pooled data approach (Models ALS77 and CFG95) is around 85% to 86%. The estimated average efficiency according to time-varying (Models K90 and KW05) and random effect (Model PL81) models is around 72%. The estimated average efficiency according to the "true fixed effect" model (computed via approach of Model CSW14) is around 91% (see Table 17).

Regarding the association of residential aged care facility characteristics with inefficiency, the average marginal effects of explanatory variables on the mean inefficiency are reported in Table 18. Specifically, based on CFG95 model, the mean inefficiency is expected to decrease by 0.26%, on average and ceteris paribus, if the occupancy rate increases by 1%. The mean inefficiency of government facilities is, on average and ceteris paribus, 11.7% higher than the mean inefficiency of not-for-profit facilities. The mean inefficiency of for-profit facilities is 2.04% less than that of not-for-profit facilities, on average and ceteris paribus. Remote facilities have the mean inefficiency being 7.3% higher than the mean inefficiency of non-remote facilities, on average and ceteris paribus. Similarly, the mean inefficiency of small facilities is 1.88% higher than that of non-small facilities, on average and ceteris paribus.

[25] Meanwhile, a limitation is that it imposes constant elasticity assumption on the model.

Regarding the quality, on average and ceteris paribus, the mean inefficiencies of Q1 and Q2 facilities are 1.9% and 1.96% lower than that of Q3 facilities, respectively. The significant positive association between efficiency and quality (or negative association between quality and inefficiency) for the log-log specification here is consistent with the linear functional form discussed in Section 4.3.1.

Table 16. Frontier estimation for logarithmic functional form, raw output

LnTC (log of total cost)	ALS77	CFG95	PL81	K90	KW05	SS84	CSW14
Frontier							
lnY (log of occupied bed days)	1.04***	1.06***	0.98***	0.96***	0.96***	0.76***	0.79***
	(0.005)	(0.005)	(0.008)	(0.007)	(0.007)	(0.011)	(0.012)
Financial year 2014/15	-0.09***	-0.1***	-0.08***	-0.18***	-0.18***	-0.09***	-0.09***
	(0.008)	(0.01)	(0.004)	(0.007)	(0.007)	(0.003)	(0.003)
Financial year 2015/16	-0.07***	-0.07***	-0.06***	-0.14***	-0.13***	-0.06***	-0.05***
	(0.008)	(0.01)	(0.004)	(0.007)	(0.006)	(0.003)	(0.003)
Financial year 2016/17	-0.04***	-0.05***	-0.04***	-0.1***	-0.08***	-0.04***	-0.03***
	(0.008)	(0.01)	(0.003)	(0.007)	(0.004)	(0.003)	(0.003)
Financial year 2017/18	-0.02***	-0.03***	-0.02***	-0.05***	-0.04***	-0.02***	-0.01***
	(0.008)	(0.01)	(0.003)	(0.005)	(0.003)	(0.003)	(0.003)
Constant	5***	4.81***	5.43***	5.65***	5.64***	8.01***	
	(0.049)	(0.05)	(0.075)	(0.071)	(0.071)	(0.115)	
Log Likelihood	1,911	1,829	4,337	4,450	4449	7,933	2,471
$\ln(\sigma_u^2)$ ***(Inefficiency - variance)***							
Constant	-3.06***	-0.15	-1.91***	-1.57***			
	(0.051)	(0.24)	(0.051)	(0.069)			
Occupancy rate		-0.03***					
		(0.003)					
For-profit		-0.25***					
		(0.09)					
Government		1.45***					
		(0.11)					
Remote		0.91***					
		(0.22)					
Small		0.23*					
		(0.12)					
Q1		-0.24**					
		(0.11)					
Q2		-0.24***					
		(0.08)					
μ_u ***(Inefficiency - mean)***							
Constant							
Occupancy rate							
For-profit							
Government							
Remote							
Small							
Q1							
Q2							
$ln(\sigma_v^2)$ ***(Residuals)***							
Constant	-4.06***	-4.08***	-5.18***	-5.26***			
	(0.042)	(0.04)	(0.023)	(0.023)			
G(t)							
t				0.04			
				(0.036)			
t^2				0.01**			
				(0.006)			
$t - \underline{t}$					-0.07***		
					(0.005)		

ALS77: Aigner, Lovell and Schmidt (1977); CFG95: Caudill, Ford, and Gropper (1995); PL81: Pitt and Lee (1981); K90: Kumbhakar (1990); KW05: Kumbhakar and Wang (2005); SS84: Schmidt and Sickles (1984); CSW14: Chen, Schmidt, and Wang (2014). See Section 4.1.
G(t): time-varying component; t: time; $\underline{t}$: the beginning period of sample.
The base case is: financial year 2018/19, not-for-profit, not remote, 30+ beds, Q3.
Standard errors in parentheses
* p < 0.10; ** p < 0.05; *** p < 0.01

Table 17. Estimated efficiency scores for logarithmic functional form, raw output

Model	Observations	Mean	Standard deviation	Min	Max	Histogram
ALS77	5,711	85.09%	7.50%	44.04%	96.76%	Figure 24
CFG95	5,711	86.03%	7.89%	37.93%	96.89%	Figure 25
PL81	5,711	72.42%	11.57%	30.22%	99.36%	Figure 26
K90	5,711	71.58%	11.86%	27.30%	99.49%	Figure 27
KW05	5,711	71.61%	11.85%	26.75%	99.48%	Figure 28
SS84	5,711	38.85%	9.18%	15.62%	100.00%	Figure 29
CSW14	5,711	91.13%	4.59%	59.09%	99.94%	Figure 30

ALS77: Aigner, Lovell and Schmidt (1977); CFG95: Caudill, Ford, and Gropper (1995); PL81: Pitt and Lee (1981); K90: Kumbhakar (1990); KW05: Kumbhakar and Wang (2005); SS84: Schmidt and Sickles (1984); CSW14: Chen, Schmidt, and Wang (2014). See Section 4.1.

Table 18. Marginal effects of explanatory variables on mean inefficiency for logarithmic functional form, raw output

	CFG95 Marginal Effects	Simple interpretation of the marginal effect
Occupancy rate	-0.0026	Lower cost inefficiency
For-profit	-0.0204	Lower cost inefficiency
Government	0.1170	Higher cost inefficiency
Remote	0.0730	Higher cost inefficiency
Small	0.0188	Higher cost inefficiency
Q1	-0.0196	Lower cost inefficiency
Q2	-0.0190	Lower cost inefficiency

CFG95: Caudill, Ford, and Gropper (1995). See Section 4.1.

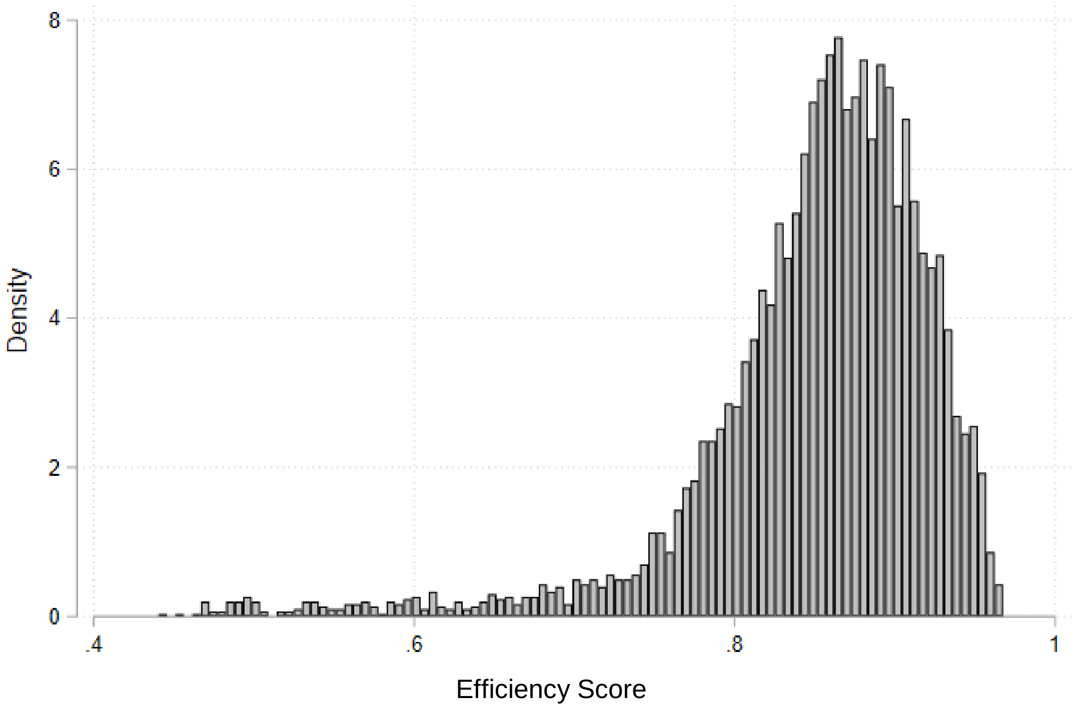

Figure 24. Histogram of estimated efficiency for ALS77 model, logarithmic functional form, raw output

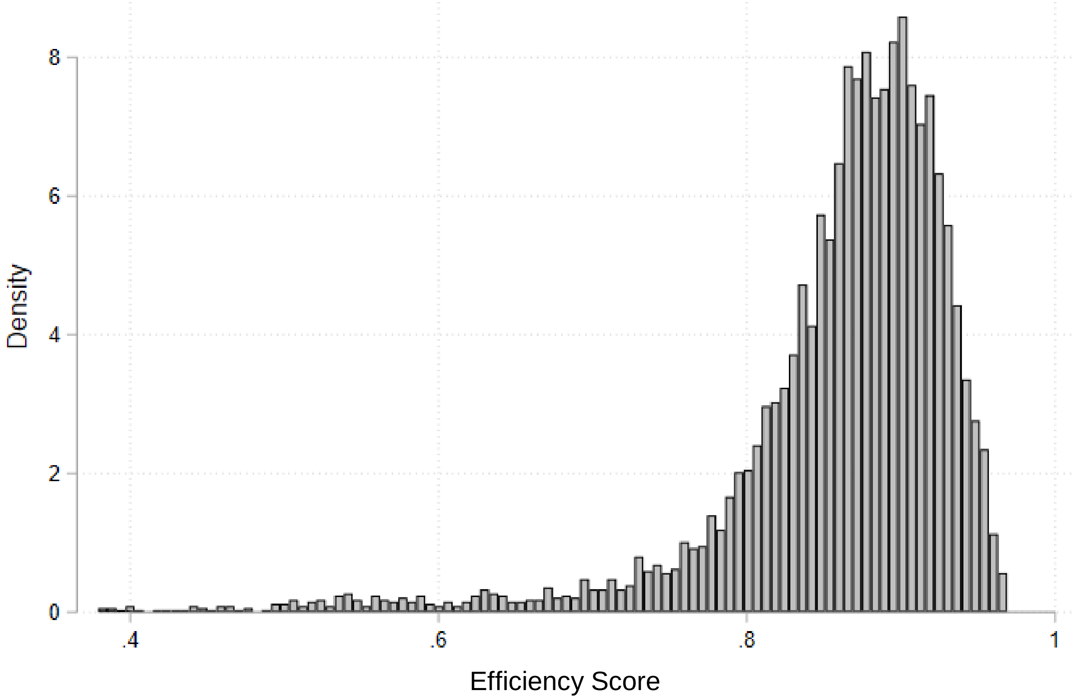

Figure 25. Histogram of estimated efficiency for CFG95 model, logarithmic functional form, raw output

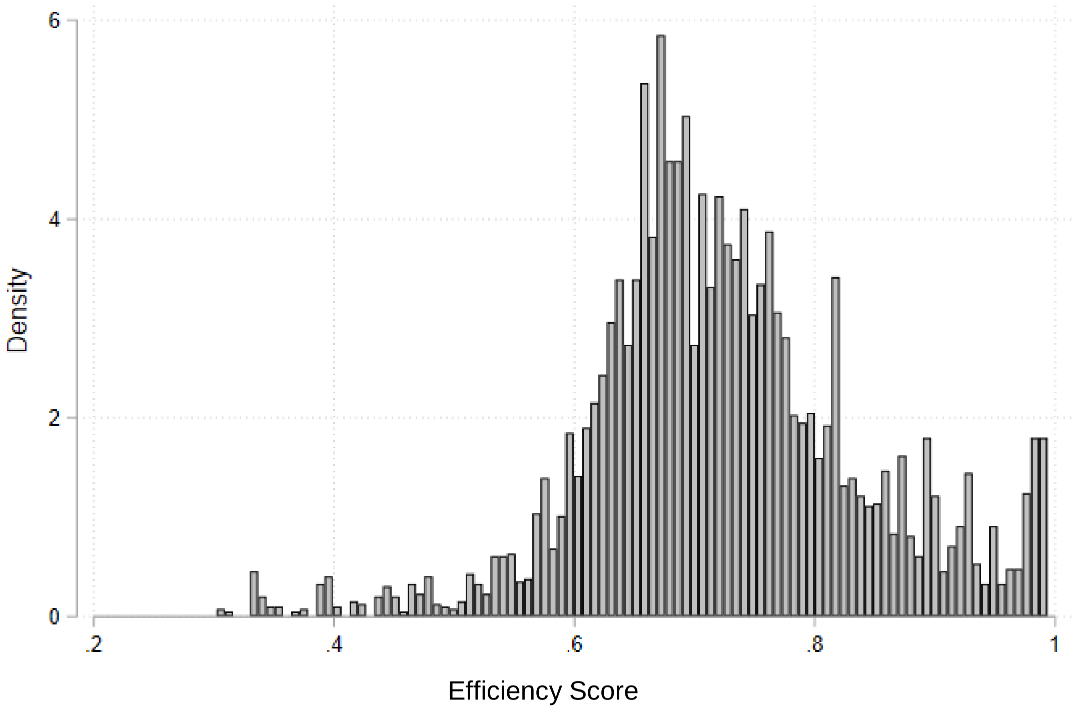

Figure 26. Histogram of estimated efficiency for PL81 model, logarithmic functional form, raw output

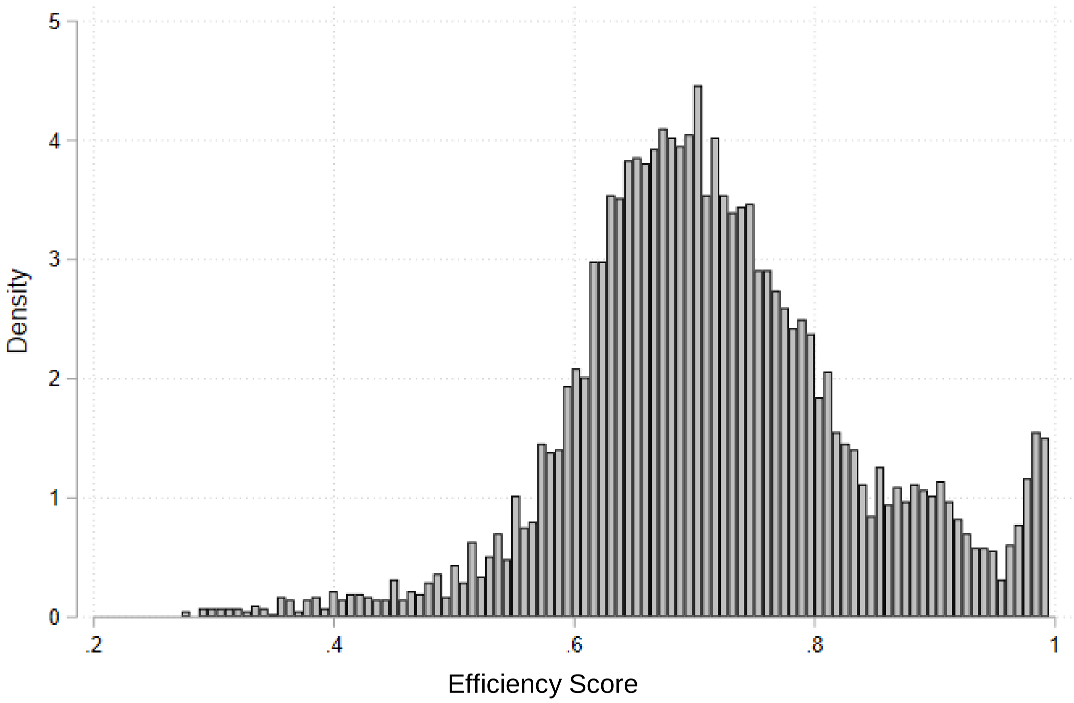

Figure 27. Histogram of estimated efficiency for K90 model, logarithmic functional form, raw output

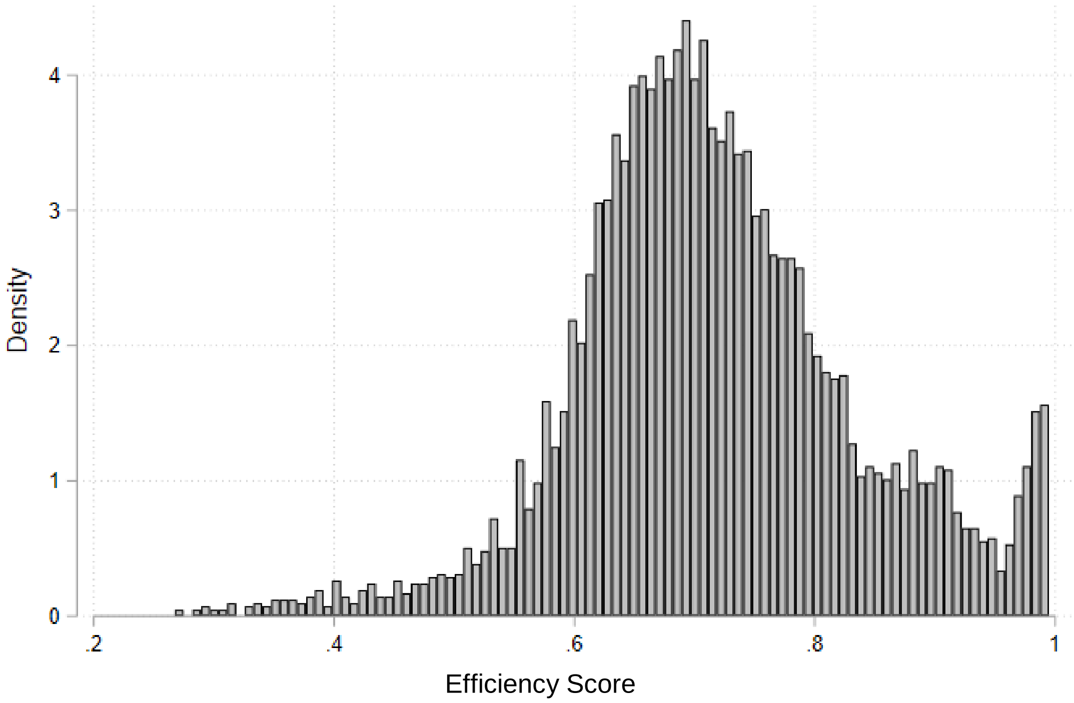

Figure 28. Histogram of estimated efficiency for KW05 model, logarithmic functional form, raw output

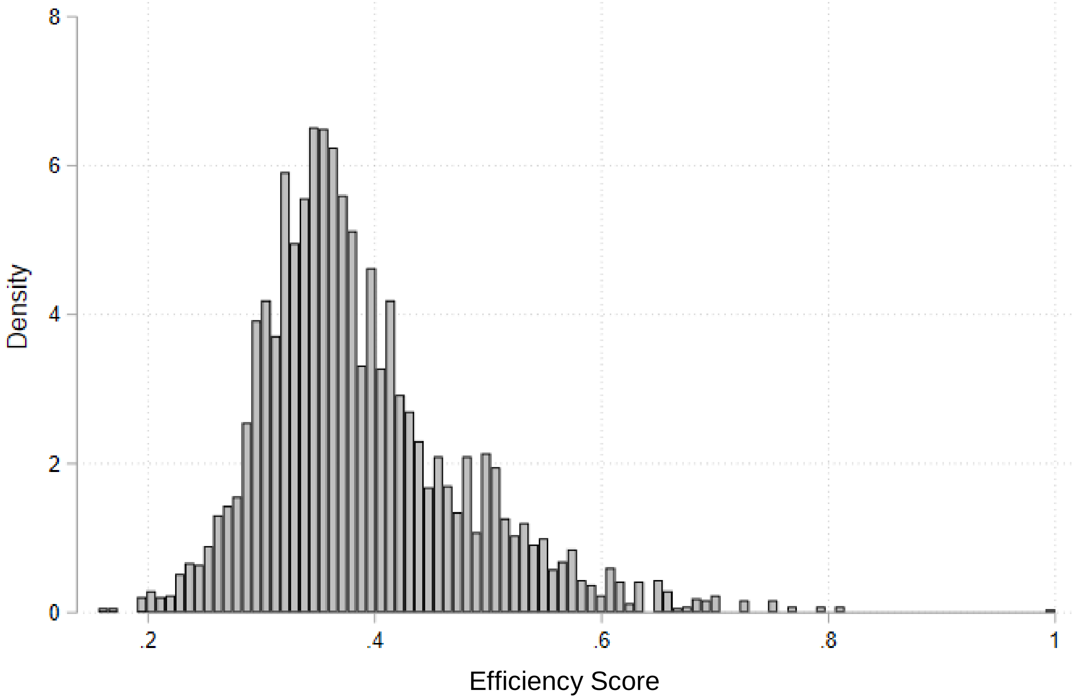

Figure 29. Histogram of estimated efficiency for SS84 model, logarithmic functional form, raw output

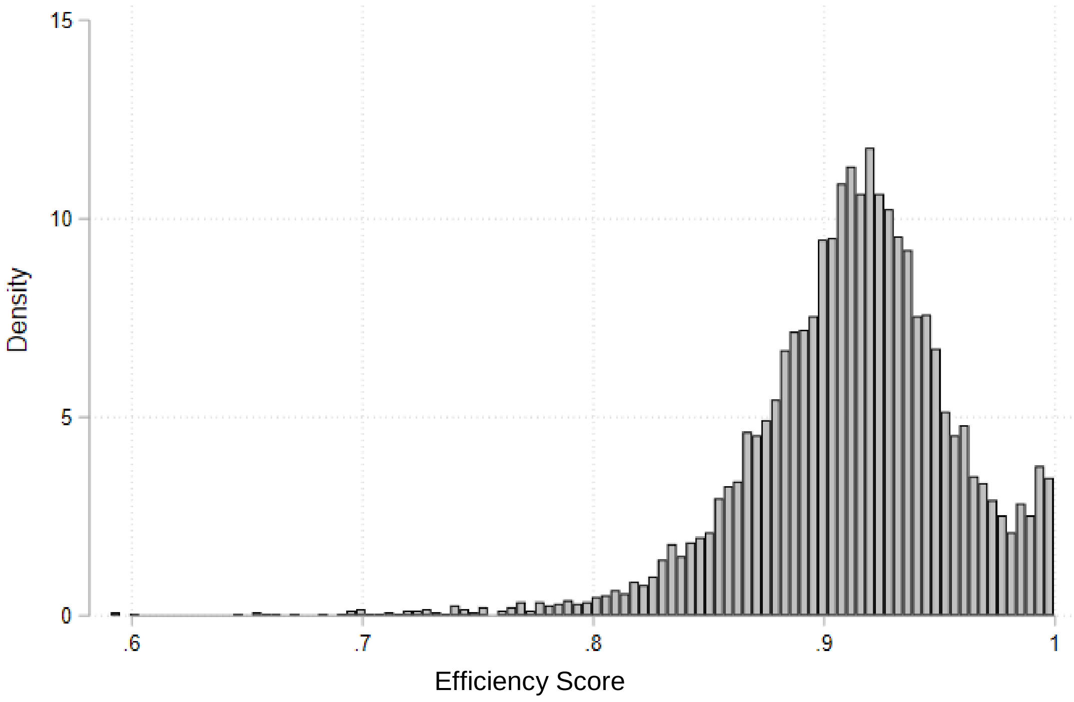

Figure 30. Histogram of estimated efficiency for CSW14 model, logarithmic functional form, raw output

4.3.2.2 Casemix adjusted output

For the logarithmic functional form with casemix adjusted output, eight models were computationally feasible as summarised in Table 19. As with the case of raw output, the estimated results for frontier coefficients are similar between the ALS77, BC95 and CFG95 (pooled data approach), between the K90 and BC92 (time-varying inefficiency models with deterministic and stochastic components), and between the SS84 and CSW14 (fixed effect approach) models. The estimated coefficients of the frontier models are consistent with the corresponding regression models in the preliminary regression analysis. A large difference is observed in the elasticity of total cost with respect to output between the pooled data approach and fixed effects approach. Again, the pooled models are likely to overestimate this coefficient due to not accounting for unobserved heterogeneity over the individuals, which to some extent is modelled by the fixed effects approach.

Regarding the estimated efficiency for the cross-sectional approach, as shown in Table 20, the histograms of estimated efficiency scores look reasonable (i.e. do not show any abnormalities or degeneracy) with the average efficiency level of around 85% to 89%. Similarly, the estimated efficiency according to the time-varying models (K90 and BC92) has an average efficiency level of around 83%.

For the fixed effects approach, the SS84 model results in a very low efficiency level (possibly due to its caveats explained above). Meanwhile, the "true fixed effects" model (via CSW14) results in a reasonable distribution of efficiency scores with the average efficiency level of around 93%.

Regarding the association of residential aged care facilities characteristics with inefficiency, the average marginal effects of explanatory variables on the mean inefficiency are reported in Table 21. From the estimated results (Table 19), we can see that the conclusions about the relationship between explanatory variables and inefficiency are qualitatively similar between the BC95 and CFG95 models. To be consistent with the discussion in the previous section, only the marginal effects according to CFG95 model are discussed.

Specifically, based on CFG95 model, the mean inefficiency is expected to decrease by 0.21%, on average and ceteris paribus, if the occupancy rate increases by 1%. The mean inefficiency of government facilities is, on average and ceteris paribus, 5.92% higher than the mean inefficiency of not-for-profit facilities. The mean inefficiency of for-profit facilities is 4.14% lower than that of not-for-profit facilities, on average and ceteris paribus. Remote facilities have the mean inefficiency being 5.49% higher than the mean inefficiency of non-remote facilities, on average and ceteris paribus. Similarly, the mean inefficiency of small facilities is 1.80% higher than that of non-small facilities, on average and ceteris paribus.

Given the potential for misinterpretation, we once again wish to remind readers that the differences between for-profit, not-for-profit and government should be interpreted with great caution as they may

(and perhaps are likely to) reflect differences in quality achieved by the different ownership types which have not been able to be distinguished within the three quality levels by the composite quality index.

Regarding the quality, the association between quality and inefficiency was found to be opposite compared to the cases discussed in previous section, on average and ceteris paribus. The mean inefficiencies of Q1 facilities are 2.83% higher than that of Q3 facilities. There is no significant evidence about the difference in mean efficiency between Q2 and Q3 facilities.

Table 19. Frontier estimation for logarithmic functional form, casemix adjusted output

LnTC (log of total cost)	ALS77	BC95	CFG95	PL81	K90	BC92	SS84	CSW14
Frontier								
lnY* (log of casemix adjusted	0.9***	0.92***	0.92***	0.85***	0.85***	0.85***	0.56***	0.56***
occupied bed days)	(0.003)	(0.003)	(0.004)	(0.005)	(0.005)	(0.005)	(0.01)	(0.009)
Financial year 2014/15	-0.03***	-0.03***	-0.03***	-0.02***	-0.06***	-0.06***	-0.05***	-0.05***
	(0.006)	(0.01)	(0.01)	(0.004)	(0.007)	(0.007)	(0.004)	(0.004)
Financial year 2015/16	-0.04***	-0.04***	-0.04***	-0.03***	-0.07***	-0.05***	-0.04***	-0.04***
	(0.006)	(0.01)	(0.01)	(0.004)	(0.007)	(0.005)	(0.003)	(0.003)
Financial year 2016/17	-0.01**	-0.01**	-0.01**	-0.01*	-0.04***	-0.02***	-0.02***	-0.01***
	(0.006)	(0.01)	(0.01)	(0.004)	(0.007)	(0.004)	(0.003)	(0.003)
Financial year 2017/18	-0.01	-0.01	-0.01	-0.004	-0.026***	-0.012***	-0.01***	-0.008***
	(0.006)	(0.01)	(0.01)	(0.004)	(0.005)	(0.004)	(0.003)	(0.003)
Constant	6.37***	6.27***	6.22***	6.92***	6.91***	6.92***	10.04***	
	(0.034)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.096)	
Log Likelihood	2,787	2,967	2,939	4,418	4,446	4,437	7,577	2,125
$\ln(\sigma_u^2)$ ***(Inefficiency – Variance)***								
Constant	-3.13***	-1.74***	-0.83***	-2.83***	-2.88***	-2.97***		
	(0.04)	(0.31)	(0.22)	(0.052)	(0.101)	(0.057)		
Occupancy rate			-0.03***					
			(0.002)					
For-profit			-0.52***					
			(0.08)					
Government			0.74***					
			(0.1)					
Remote			0.69***					
			(0.2)					
Small			0.23**					
			(0.1)					
Q1			0.36***					
			(0.1)					
Q2			-0.08					
			(0.07)					
μ_u ***(Inefficiency – Mean)***								
Constant		0.79***						
		(0.19)						
Occupancy rate		-0.02***						
		(0.006)						
For-profit		-0.6***						
		(0.2)						
Government		0.58***						
		(0.18)						
Remote		0.63***						
		(0.22)						
Small		0.12						
		(0.1)						
Q1		0.36***						
		(0.14)						
Q2		-0.02						
		(0.08)						
$ln(\sigma_v^2)$ ***(Residuals)***								
Constant	-4.89***	-4.64***	-4.87***	-4.97***	-4.98***	-4.98***		
	(0.056)	(0.05)	(0.06)	(0.022)	(0.022)	(0.022)		
$G(t)$								
t					-0.18**			
					(0.074)			

LnTC (log of total cost)	ALS77	BC95	CFG95	PL81	K90	BC92	SS84	CSW14
t^2					0.05***			
					(0.013)			
$t - T$						-0.04***		
						(0.007)		

ALS77: Aigner, Lovell and Schmidt (1977); BC95: Battese and Coelli (1995); CFG95: Caudill, Ford, and Gropper (1995); PL81: Pitt and Lee (1981); K90: Kumbhakar (1990); BC92: Battese and Coelli (1992); SS84: Schmidt and Sickles (1984); CSW14: Chen, Schmidt, and Wang (2014). See Section 4.1.
G(t): time-varying component; t: time; T: the end period of the sample.
The base case is: financial year 2018/19, not-for-profit, not remote, 30+ beds, Q3.
Standard errors in parentheses
* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$

Table 20. Estimated efficiency scores for logarithmic functional form, casemix adjusted output

Model	Observations	Mean	Standard deviation	Minimum	Maximum	Histogram
ALS77	5,711	85.48%	8.45%	48.55%	97.96%	Figure 31
BC95	5,711	89.17%	7.84%	49.43%	97.76%	Figure 32
CFG95	5,711	86.01%	8.51%	47.52%	97.83%	Figure 33
SS84	5,711	35.34%	9.23%	16.07%	100.00%	Figure 34
PL81	5,711	82.64%	9.86%	43.29%	99.18%	Figure 35
K90	5,711	82.79%	9.89%	42.38%	99.27%	Figure 36
BC92	5,711	82.72%	9.89%	41.62%	99.23%	Figure 37
CSW14	5,711	93.16%	3.15%	66.52%	99.51%	Figure 38

ALS77: Aigner, Lovell and Schmidt (1977); BC95: Battese and Coelli (1995); CFG95: Caudill, Ford, and Gropper (1995); SS84: Schmidt and Sickles (1984); PL81: Pitt and Lee (1981); K90: Kumbhakar (1990); BC92: Battese and Coelli (1992); CSW14: Chen, Schmidt, and Wang (2014). See Section 4.1.

Table 21. Marginal effects of explanatory variables on mean inefficiency for logarithmic functional form, casemix adjusted output

	BC95 Marginal Effects	CFG95 Marginal Effects	Simple interpretation of the marginal impact
Occupancy rate	-0.0017	-0.0021	Lower cost inefficiency
For-profit	-0.0458	-0.0414	Lower cost inefficiency
Government	0.0440	0.0592	Higher cost inefficiency
Remote	0.0478	0.0549	Higher cost inefficiency
Small	0.0092	0.0180	Higher cost inefficiency
Q1	0.0275	0.0283	Higher cost inefficiency
Q2	-0.0017	-0.0062	Lower cost inefficiency

BC95: Battese and Coelli (1995); CFG95: Caudill, Ford, and Gropper (1995). See Section 4.1.
The base case is: financial year 2018/19, not-for-profit, not remote, 30+ beds, Q3.

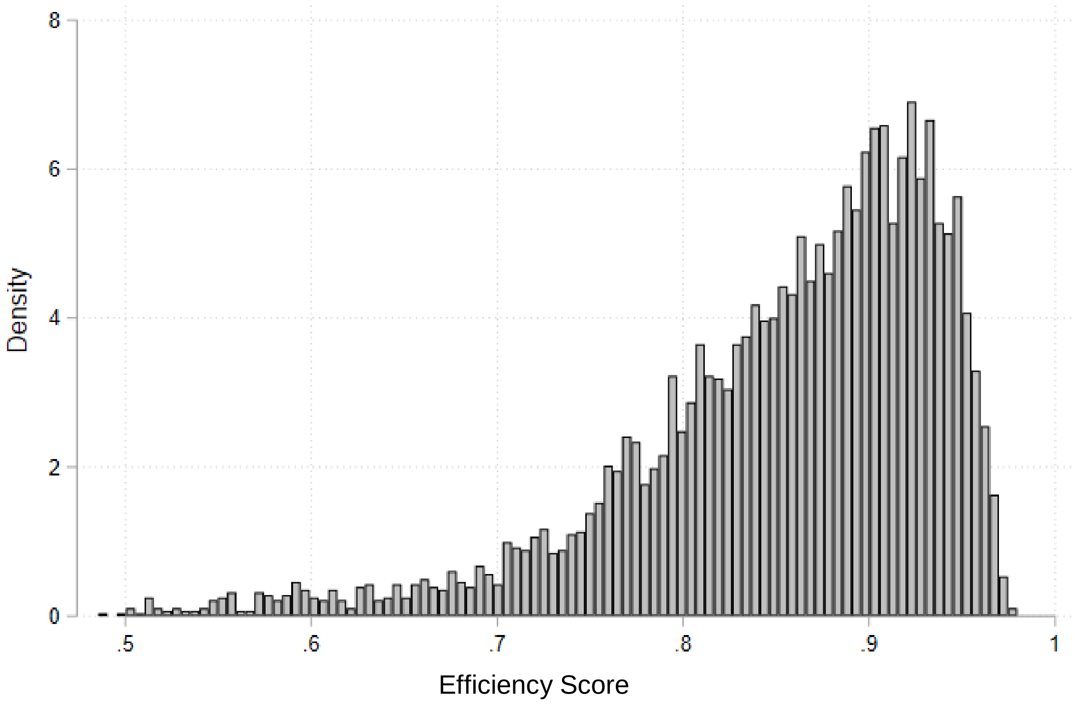

Figure 31. Histogram of estimated efficiency for ALS77 model, logarithmic functional form, casemix adjusted output

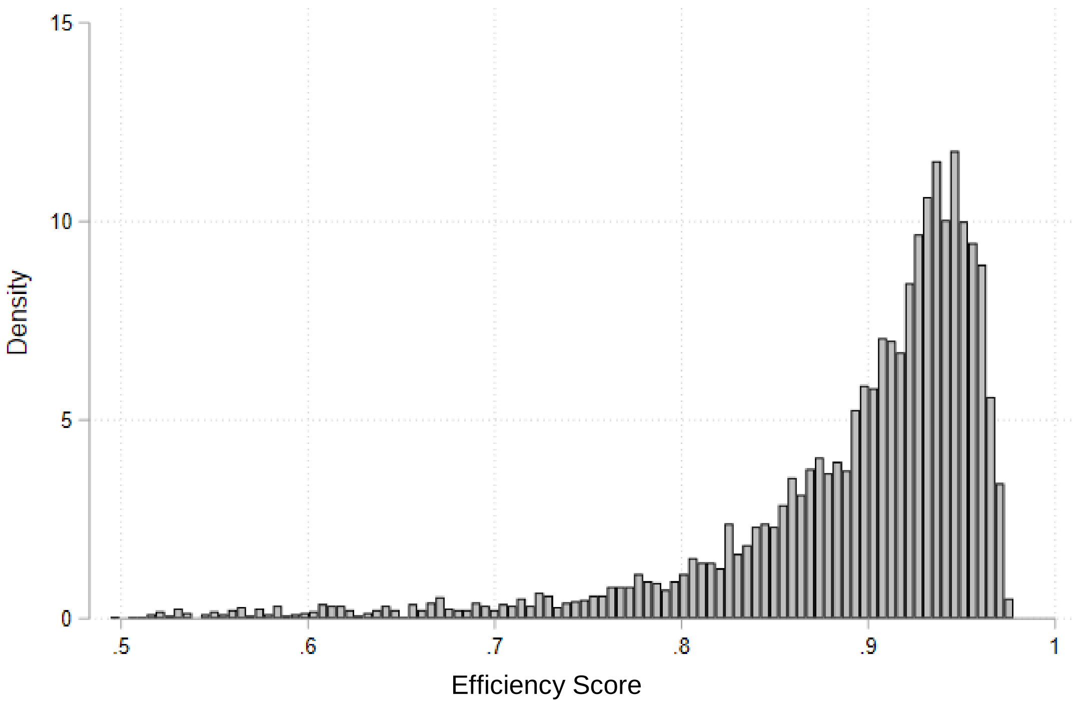

Figure 32 Histogram of estimated efficiency for BC95 model, logarithmic functional form, casemix adjusted output

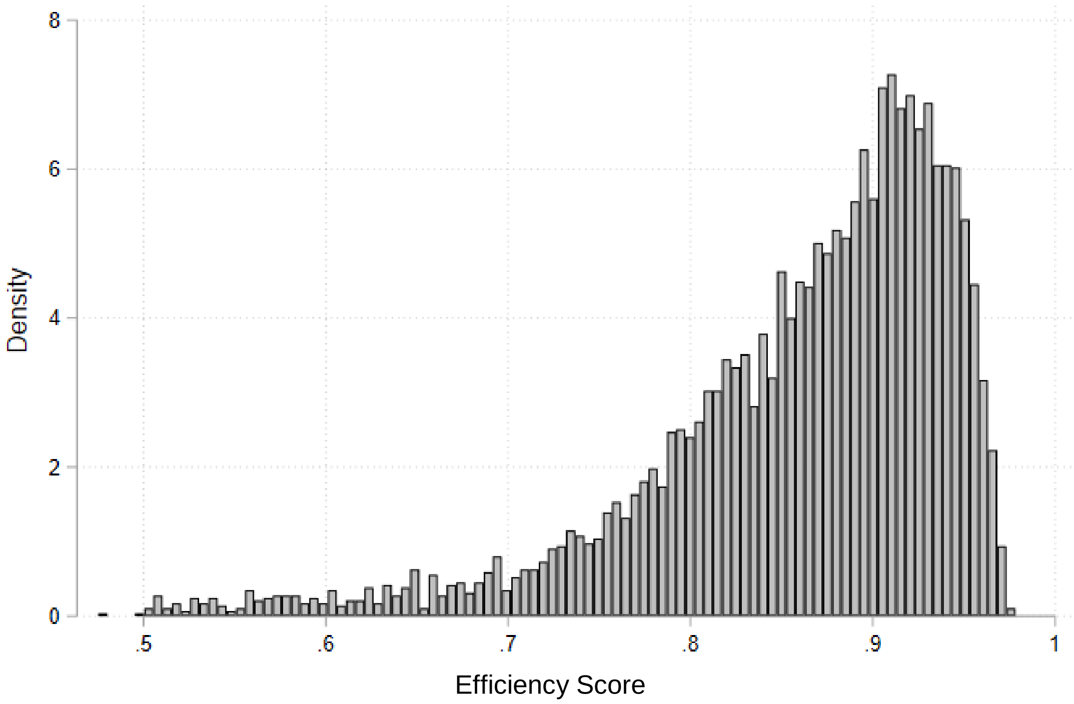

Figure 33. Histogram of estimated efficiency for CFG95 model, logarithmic functional form, casemix adjusted output

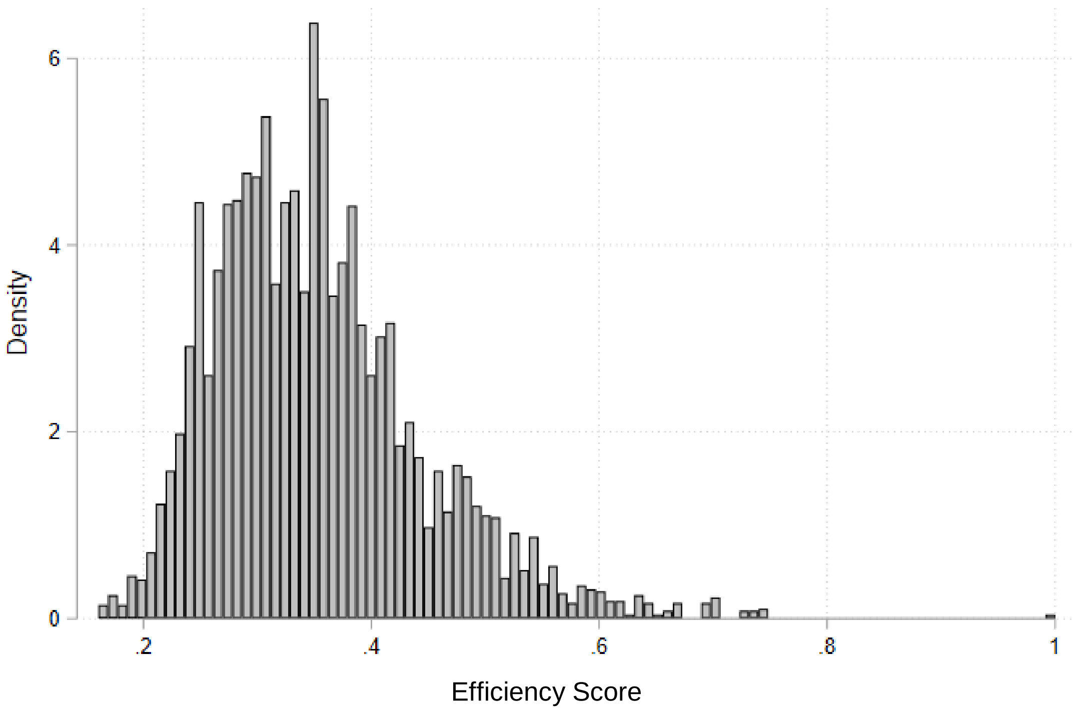

Figure 34. Histogram of estimated efficiency for SS84 model, logarithmic functional form, casemix adjusted output

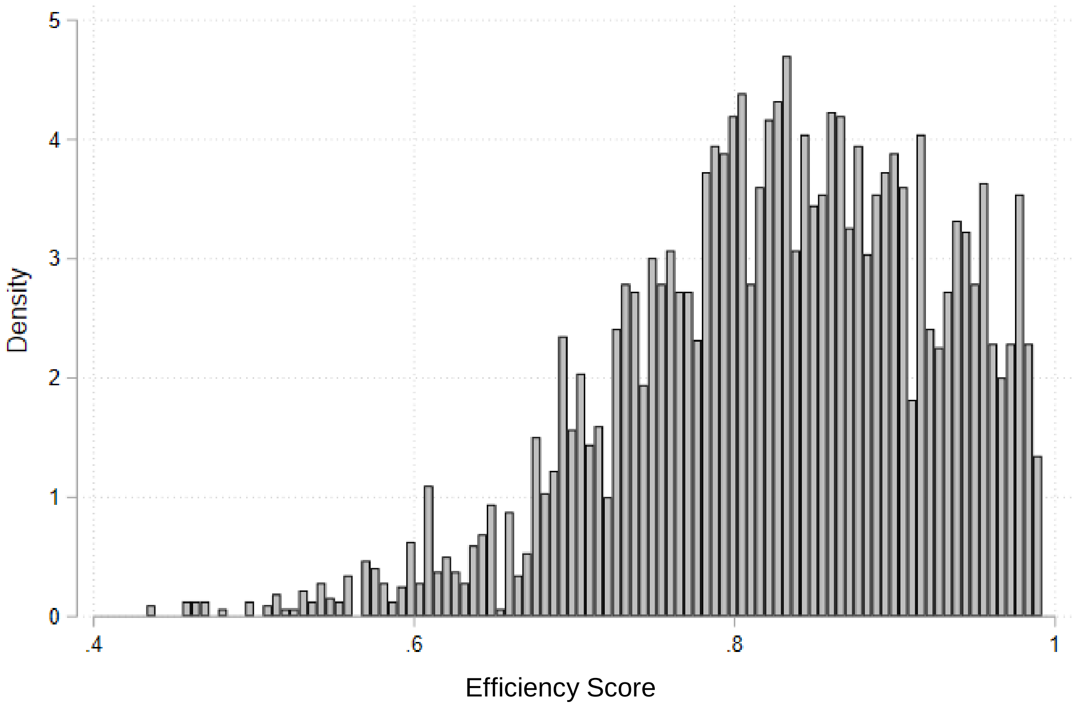

Figure 35. Histogram of estimated efficiency for PL81 model, logarithmic functional form, casemix adjusted output

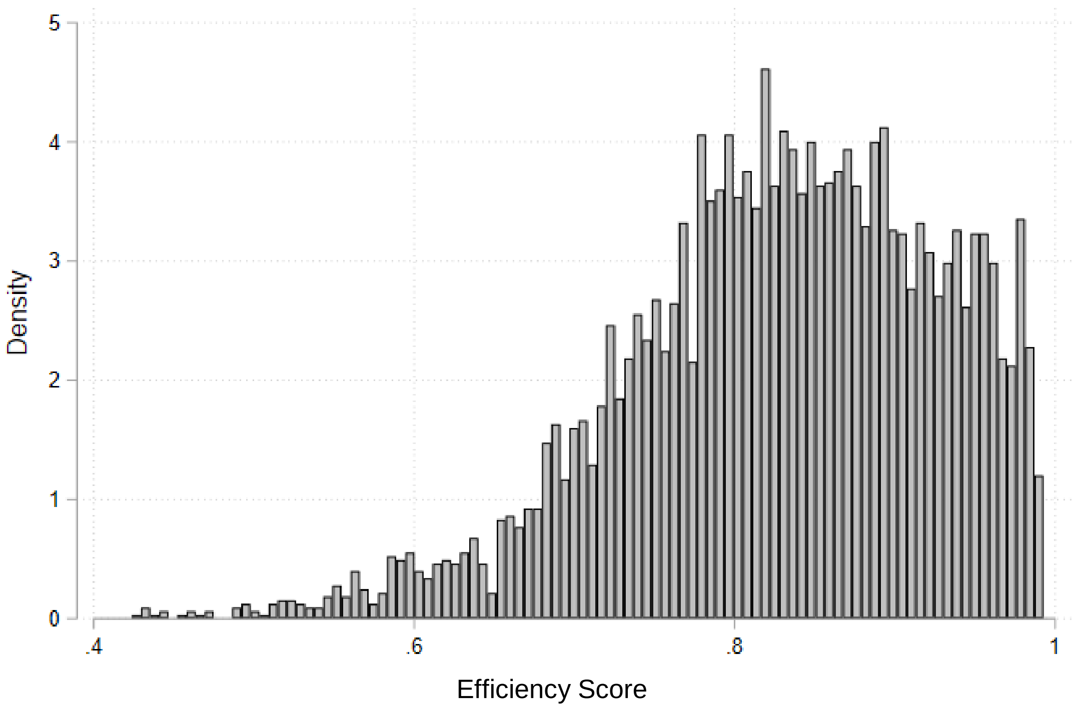

Figure 36. Histogram of estimated efficiency for K90 model, logarithmic functional form, casemix adjusted output

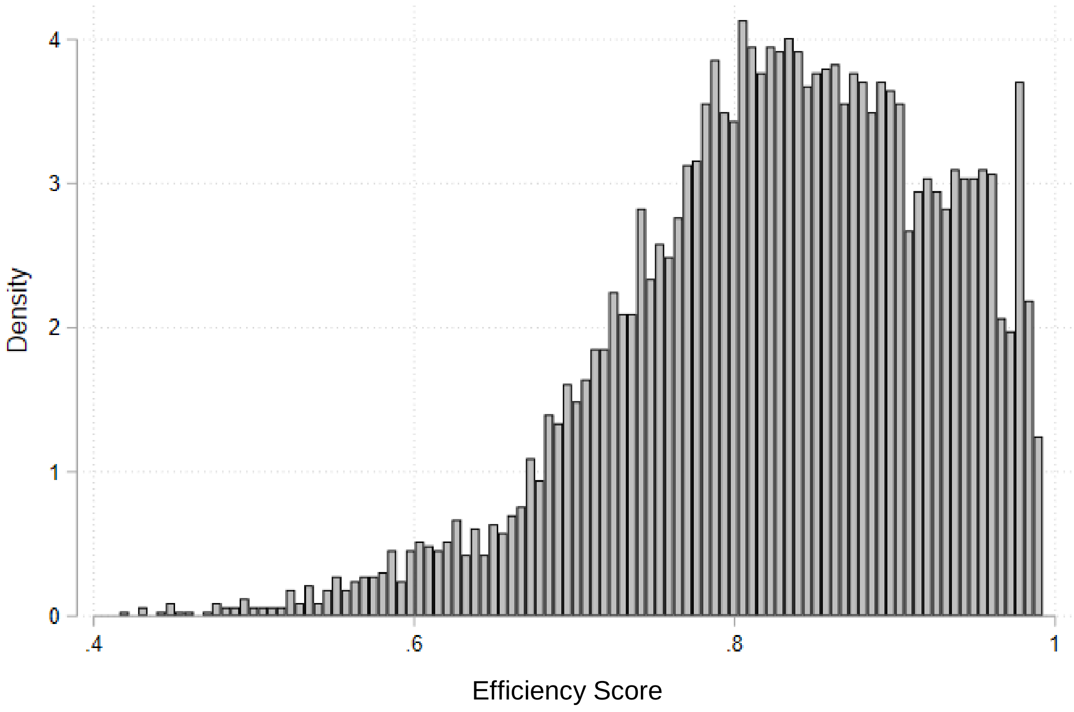

Figure 37. Histogram of estimated efficiency for BC92 model, logarithmic functional form, casemix adjusted output

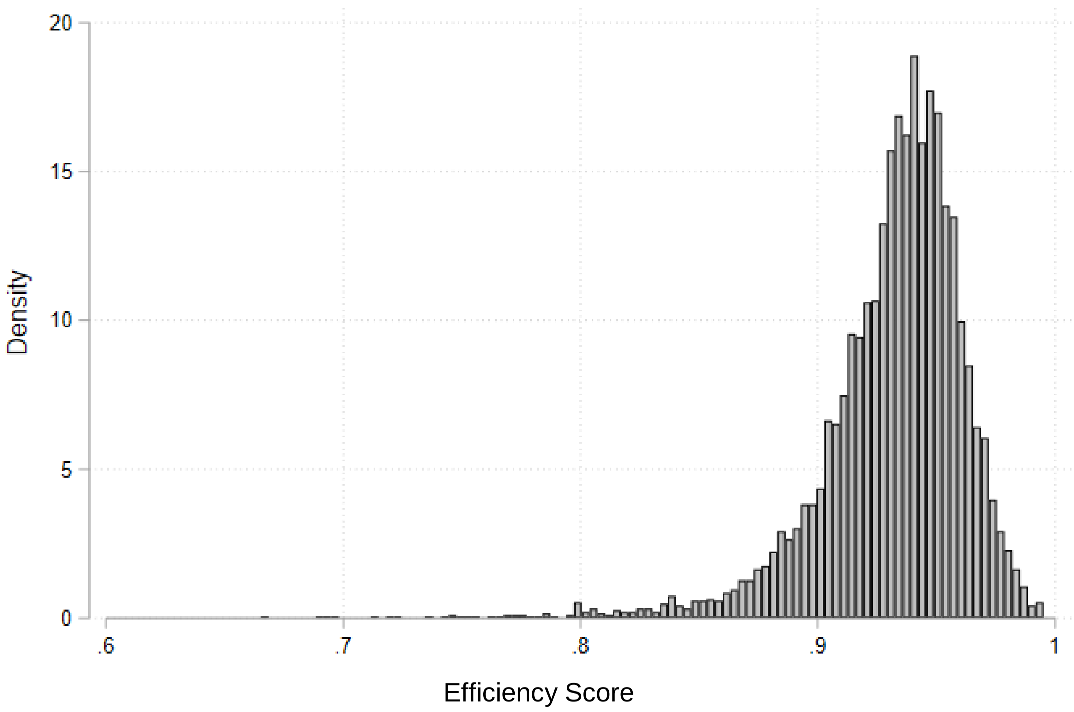

Figure 38. Histogram of estimated efficiency for CSW14 model, logarithmic functional form, casemix adjusted output

4.4 Concluding remarks

Performing stochastic frontier analysis for a large data set with wide heterogeneity such as in this project is particularly challenging. Due to the relatively complicated log-likelihood functions needed to optimise, most of the estimation procedures required extensive computational power and time to try various specifications. Some models with desirable theoretical properties still could not converge even after many hours of running.

As a result, to choose a suitable model for our data, a variety of methods were tried starting from the simplest model (i.e. the cross-sectional ALS77 model) and then several more complicated models (e.g. the models with persistent and time-varying inefficiency). Some of the models were infeasible to estimate with the available software. The feasible models were the ALS77 (equations 1-4), BC95 (equations 11-15), CFG95 (equations 16-20), SS84 (equations 21-22), PL81 (equations 25-28), K90, BC92 and KW05 (equations 29-36), and CSW14 (equations 37-40) models.

The feasible models were compared with the corresponding regression models discussed in the preliminary regression analysis (which did not model inefficiency) and the distributions of estimated (in)efficiency were examined. The estimated coefficients of frontier models (particularly for the main variable of interest, output) were consistent with the corresponding regression models, which indicates some robustness of the conclusions.

To investigate the sensitivity of stochastic frontier models with respect to the parametric assumptions on the functional form of the frontier and on the distribution of the statistical noise, the non-parametric least square methods for stochastic frontier models, developed by Simar, Van Keilegom and Zelenyuk (2017) (hereafter SVKZ) were used. This was done to estimate the cost frontiers and inefficiency of the industry and compare the results with those obtained using parametric stochastic frontier models. Based on the comparison (see details in *Appendix B* of this *Technical Supplement Report 2*, the stochastic frontier models applied so far are considered robust to the parametric assumption, especially for the log-log transformation with casemix adjusted output, where both average inefficiency and the shape of the frontier estimated from parametric stochastic frontier models are very close to those estimated using the non-parametric model.

Among the feasible models, it appears that the flexible log-log function estimated using ALS77 is the most appropriate for the context of this project. The estimates from final model suggests that the average inefficiency in the industry is at about 12%. Other models, involving different assumptions, showed higher or lower levels of inefficiency.

5. Stochastic frontier modelling for disaggregated costs

5.1 Model specifications

The association between quality levels, inefficiency and different cost categories (e.g. direct care, hotelling, accommodation, administration, and other costs) is of particular interest to the stakeholders of this project. It was believed that Q1 facilities had a higher efficient cost of care than those of lower-quality facilities, due to a better workforce, working environment and clinical governance. Moreover, for facilities of different quality levels, the costs of different categories might have different relationships with output. For example, when output increases, Q1 facilities might spend more on hotelling services than Q3 facilities. As a result, in addition to the total cost frontier, separate frontiers for each cost category, with the quality indices included in the frontier functions, were estimated. The estimated results are presented in this section.

In this section, flexible log-log specification with the casemix-adjusted output (Y^*) was applied. The casemix adjustment accounts for <u>some</u> of the differences in resource use to provide care to residents of different levels of frailty (severity). The log-log transformation helps to reduce the impact of potential heteroskedasticity in the data associated with casemix and the sparsity at the tails (of the cost and output distributions) as the log-log transformation shrinks data scale. This strategy appears to have worked well in the context of this project.[26] As discussed in the previous section, among different specifications, with the log-log transformation and the casemix adjustment, stochastic frontier models have a desirable robustness property with respect to the parametric assumption, where both average inefficiency and the shape of the frontiers estimated from stochastic frontier models are very close to those estimated using the non-parametric model.[27]

For each cost category, the right-hand side of the cost frontier includes output (in log form), the four remaining cost categories[28] (all in log form), year dummies and together with quality dummies (i.e. Q1 and Q2) and their interaction terms with output. The flexible log-log form also includes in the frontier function the quadratic term of log output and its interaction terms with quality dummies. For comparison, the total cost frontier is estimated here with the similar specification. Specifically, the right-hand side of the total cost frontier includes output (in log form), the quadratic term of log output, quality dummies and their interaction terms with log output and quadratic term of log output.[29]

It is hypothesised that certain characteristics of facilities influence the service provision (i.e. production of residential aged care services), and are associated with whether or not a facility can operate at its optimal productivity. To test this hypothesis, variables that capture characteristics and operational standard of facilities were included as exogenous determinants of the inefficiency. These variables include occupancy (*occupancy rate)*, remoteness (*remote*), size (*small*), type (*for-profit* and *government*), and quality (*Q1 and Q2*).[30] All these variables are the same variables as in previous models explored above.

5.2 Empirical results

Due to a limitation in data reporting practices, the available data for the disaggregated cost analysis contains less observations than that used for the total cost analysis (5,220 observations vs. 6,188 observations) (accounting for about 37% of the whole industry). For more details, refer to the *Main report, Appendix B: Direct care cost and workforce data*.

[26] Meanwhile, a limitation is that it imposes constant elasticity assumption on the model.

[27] In the previous section, we utilised the non-parametric least square methods for stochastic frontier models developed by Simar, Van Keilegom and Zelenyuk (2017) (hereafter SVKZ) to investigate the sensitivity of the estimated frontiers and inefficiency of SFA models with respect to the parametric assumptions on the functional form of the frontier and on the distribution of the statistical noise.

[28] Note that not including the other cost categories in the model may introduce the bias to other estimates (unless there is no correlation between it and the other regressors) and we do not recommend it, because, indeed, from simple economic reasoning we know that the other cost categories are important in the production process and therefore better be included in the model.

[29] To examine the appropriateness for including quadratic term of log output, we compared the results (estimated efficiency and frontier) from models with and without this term with the most flexible SFA we know so far (i.e., SVKZ). It seems to be that the model with the quadratic term of log output is the closest to SVKZ for this data. Moreover, the results from this model seem to make a lot of sense on intuitive grounds as discussed below.

[30] Q1 represents RACFs in the highest quality group, and Q2 represents RACFs in the medium high and medium quality groups.

Following the same procedure as with the total cost analysis, the observations that belong to facilities which have only single-year record (160 out of 5,220 observations) were dropped. Observations in 2.5% of each tail of the distributions of the average cost (per casemix adjusted bed day) of the five cost categories (832 out of 5,220 observations) were excluded. As a result, the sample for the disaggregated cost analysis includes 4,228 observations (accounting for about 31% of the whole industry).

Before estimating the stochastic frontier models, the **residual analysis** was performed to examine the existence of inefficiency in each cost category and total cost. The results suggest that the **inefficiency is associated with total cost and direct care cost**, but **not** in the other four cost categories. This is confirmed by the posterior likelihood ratio test after estimating the ALS77 model for each of the four cost categories, where no significant statistical evidence about the existence of inefficiency was found. The evidence that other cost categories are not statistically associated with inefficiency (when conditioned on other factors), suggests that most (if not all) the inefficiency in the total cost is due to the direct care cost.

Furthermore, for the four cost categories, looking at the estimated results (Table 22)[31], we also do not find significant statistical evidence about the association between these cost categories and the quality categories of facilities.

Table 22. Frontier estimation for four non-care cost categories (ALS77 model)

	Ln of TC Direct Care	**Ln of TC Hotelling**	**Ln of TC Accommodation**	**Ln of TC Administration**	**Ln of TC Others**
Frontier					
Ln of casemix-adjusted OBD	1.18*** (0.33)	0.19 (0.42)	1.87*** (0.71)	0.83 (0.65)	-1.99 (1.21)
Squared of ln of casemix-adjusted OBD	-0.02 (0.02)	0.01 (0.02)	-0.07** (0.04)	-0.01 (0.03)	0.11* (0.06)
Q1	5.27** (2.27)	0.17 (2.88)	4.75 (4.88)	-2.24 (4.51)	5.07 (8.36)
Q2	3.80** (1.75)	-0.31 (2.22)	6.24* (3.76)	4.42 (3.48)	-6.41 (6.45)
Q1 * Ln of casemix-adjusted OBD	-1.09** (0.46)	-0.03 (0.59)	-0.85 (1.00)	0.70 (0.92)	-1.29 (1.71)
Q2 * Ln of casemix-adjusted OBD	-0.77** (0.35)	0.07 (0.44)	-1.23* (0.75)	-0.80 (0.69)	1.32 (1.29)
Q1 x Squared of Ln of casemix-adjusted OBD	0.06** (0.02)	0.00 (0.03)	0.04 (0.05)	-0.05 (0.05)	0.08 (0.09)
Q2 x Squared of Ln of casemix-adjusted OBD	0.04** (0.02)	-0.00 (0.02)	0.06 (0.04)	0.04 (0.03)	-0.07 (0.06)
Financial Year 14-15	-0.07*** (0.01)	0.01 (0.01)	0.00 (0.02)	-0.04*** (0.02)	-0.09*** (0.03)
Financial Year 15-16	-0.04*** (0.01)	0.01 (0.01)	-0.01 (0.02)	-0.04** (0.02)	-0.11*** (0.03)
Financial Year 16-17	-0.02*** (0.01)	0.01 (0.01)	-0.04** (0.02)	-0.05*** (0.01)	0.04 (0.03)
Financial Year 17-18	-0.01 (0.01)	0.01 (0.01)	-0.06*** (0.02)	-0.03* (0.01)	0.06** (0.03)
Ln of TC Direct care		0.32*** (0.02)	0.12*** (0.03)	-0.04 (0.03)	0.23*** (0.06)
Ln of TC Hotelling	0.19*** (0.01)		0.22*** (0.03)	0.11*** (0.02)	0.44*** (0.04)
Ln of TC Accommodation	0.03*** (0.01)	0.08*** (0.01)		0.17*** (0.01)	-0.02 (0.03)

[31] For these, because there is no significant inefficiency, the cost frontier also coincides with the average cost function.

	Ln of TC Direct Care	Ln of TC Hotelling	Ln of TC Accommodation	Ln of TC Administration	Ln of TC Others
Ln of TC Administration	-0.01* (0.01)	0.05*** (0.01)	0.20*** (0.02)		0.10*** (0.03)
Ln of TC Others	0.02*** (0.00)	0.05*** (0.01)	-0.01 (0.01)	0.03*** (0.01)	
Constant term (intercept)	2.36 (1.65)	3.46* (2.10)	-5.66 (3.57)	2.34 (3.28)	10.50* (6.08)
Uσ inefficiency component					
Constant term	-4.36*** (0.31)	-14.18 (484.31)	-14.54 (1562.80)	-13.03 (289.92)	-13.66 (873.32)
Vσ statistical noise (residuals)					
Constant term	-3.92*** (0.07)	-3.23*** (0.02)	-2.17*** (0.02)	-2.33*** (0.02)	-1.10*** (0.02)
Number of observation	4228	4228	4228	4228	4228
Akaike Information Criteria (AIC)	-3657.93	-1605.49	2841.22	2184.43	7402.27
Bayesian Information Criteria (BIC)	-3537.29	-1484.85	2961.86	2305.07	7522.91

ln: natural logarithm; OBD: occupied bed days; QI: quality index; TC: total cost
ALS77: Aigner, Lovell and Schmidt (1977). See Section 4.1. The base case is: financial year 2018/19, Q3.
* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$

For the direct care cost and total cost, all the models discussed in Section 4.1 were applied to our data. However, the optimisation algorithms in the available software for some models did not converge after many hours of running and thus were terminated. As a result, while we tried all the models listed above, the results are reported only for cross-sectional models (e.g., ALS77 and CFG95)[32] that were feasible in terms of estimation.[33]

Looking at the estimated results (Table 23) and (more intuitively) at the graphs of estimated frontiers (based on ALS77 models, in logarithms) (Figure 39 and Figure 40), we can see that the association between quality and the efficient care cost (as well as efficient total cost) is different at the different level of output. On average and ceteris paribus, Q1 and Q2 facilities tend to have higher efficient cost than Q3 facilities at low level of output, as is expected. (Note that the difference looks very small, due to the logarithmic scale.) Moreover, when the output increases, initially the marginal efficient cost of Q3 facilities is higher than that of Q2 and Q1 facilities, resulting in a higher efficient cost of Q3 facilities at medium level of output. When the output increases further, the marginal efficient cost of Q1 and Q2 facilities turns out to be higher than that of Q3 facilities. As a result, the efficient cost of Q1 and Q2 facility is higher than that of Q3 facility at high level of output.

Regarding the estimated efficiency (Table 24), the average efficiency of the industry in direct care cost is around 91% and the average efficiency in the total cost is around 88%. Another useful measure of the efficiency of an industry is the aggregate efficiency, defined as the weighted mean of individual efficiencies using the cost shares of individual facilities in the industry as the weights. As compared to the simple average of efficiency, the aggregate efficiency is useful because it takes the relative economic importance (e.g., size) of individual facilities in the industry into account.[34] The results reported in Table 25 show that the aggregate efficiency of the industry in direct care cost is also around 91% and the aggregate efficiency in the total cost is also around 88%. Thus, we can see that the simple average of efficiency and aggregate (weighted) efficiency estimates tell the same story about efficiency of the industry, adding to the robustness of related conclusions.

The estimated efficiency again confirms that most of the inefficiency in cost is due to the direct care cost. The estimated efficiencies are similar to the results from the true fixed effects panel data model (without quality)

[32] CFG95 and BC95 are two different alternatives for modelling the inefficiency on exogenous explanatory variables. To be consistent with previous section, here we focus our discussion on CFG95 model.

[33] Another model that was feasible is SS84, which does not require maximum likelihood optimisation, but as discussed in the previous section, this model is not reliable in term of estimated (in)efficiency due to its caveat, so we do not report its result here. However, it is worth mentioning that the coefficients of SS84 are estimated via fixed effects and qualitatively similar to coefficients of cross-sectional models reported in this section. This is an indication of the robustness of the results from the cross-sectional models.

[34] For theoretical justifications, see Fare and Zelenyuk (2003) and Simar and Zelenyuk (2007, 2018, 2020).

discussed in the previous section. The association between facility characteristics and cost inefficiency are presented in Table 26 together with the estimated marginal effects.

Table 23. Frontier estimation for care cost and total cost

Log of	ALS77 Direct care	CFG95 Direct care	ALS77 Total cost	CFG95 Total cost
Frontier				
lnY (casemix adjusted occupied bed days)	1.18***	1.84***	1.11***	1.14***
	(0.33)	(0.36)	(0.26)	(0.26)
$(lnY)^2$	-0.02	-0.05***	-0.01	-0.01
	(0.02)	(0.02)	(0.01)	(0.01)
Log of Accommodation cost	0.03***	0.03***	-	-
	(0.01)	(0.01)		
Log of Hotel cost	0.19***	0.19***	-	-
	(0.01)	(0.01)		
Log of Administration cost	-0.01*	-0.01	-	-
	(0.01)	(0.01)		
Log of other cost	0.02***	0.02***	-	-
	(0.004)	(0.004)		
Financial year 2014/15	-0.07***	-0.07***	-0.06***	-0.07***
	(0.01)	(0.01)	(0.01)	(0.01)
Financial year 2015/16	-0.04***	-0.04***	-0.04***	-0.04***
	(0.01)	(0.01)	(0.01)	(0.01)
Financial year 2016/17	-0.02***	-0.02***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.01)	(0.01)
Financial year 2017/18	-0.01	-0.01*	-0.01**	-0.01**
	(0.01)	(0.01)	(0.01)	(0.01)
Q1	5.27**	6.46**	4.5**	5.27***
	(2.27)	(2.72)	(1.85)	(1.85)
Q2	3.8**	4.77**	3.82***	4.26***
	(1.75)	(1.96)	(1.41)	(1.36)
lnY x Q1	-1.09**	-1.34**	-0.89**	-1.05***
	(0.46)	(0.55)	(0.38)	(0.38)
lnY x Q2	-0.77**	-0.96**	-0.76***	-0.85***
	(0.35)	(0.39)	(0.28)	(0.27)
$(lnY)^2$ x Q1	0.06**	0.07**	0.04**	0.05***
	(0.02)	(0.03)	(0.02)	(0.02)
$(lnY)^2$ x Q2	0.04**	0.05**	0.04***	0.04***
	(0.02)	(0.02)	(0.01)	(0.01)
Constant	2.36	-1.17	5.31***	5.13***
	(1.65)	(1.84)	(1.33)	(1.29)
Log Likelihood	1,848	1,948	2,621	2,681
$\boldsymbol{ln(\sigma_u^2)}$ ***(Inefficiency – Variance)***				
Constant	-4.36***	-1.65***	-3.64***	-1.03***
	(0.31)	(0.6)	(0.06)	(0.4)
Occupancy rate		-0.03***		-0.03***
		(0.01)		(0.004)
For-profit		-0.54**		-0.94***
		(0.22)		(0.15)
Government		2.05***		0.48**
		(0.27)		(0.23)
Remote		1.43***		1.01***
		(0.32)		(0.27)
Small		0.84***		-0.51**
		(0.22)		(0.22)
Q1		1.45***		0.42**
		(0.37)		(0.19)
Q2		0.23		0.2
		(0.33)		(0.15)
$\boldsymbol{ln(\sigma_v^2)}$ ***(Inefficiency – Mean)***				
Constant	-3.92***	-4.02***	-4.84***	-4.81***
	(0.07)	(0.05)	(0.06)	(0.06)

ALS77: Aigner, Lovell and Schmidt (1977); CFG95: Caudill, Ford, and Gropper (1995). See Section 4.1.
The base case is: financial year 2018/19, not-for-profit, not remote, 30+ beds, Q3.
Standard errors in parentheses
* p < 0.10; ** p < 0.05; *** p < 0.01

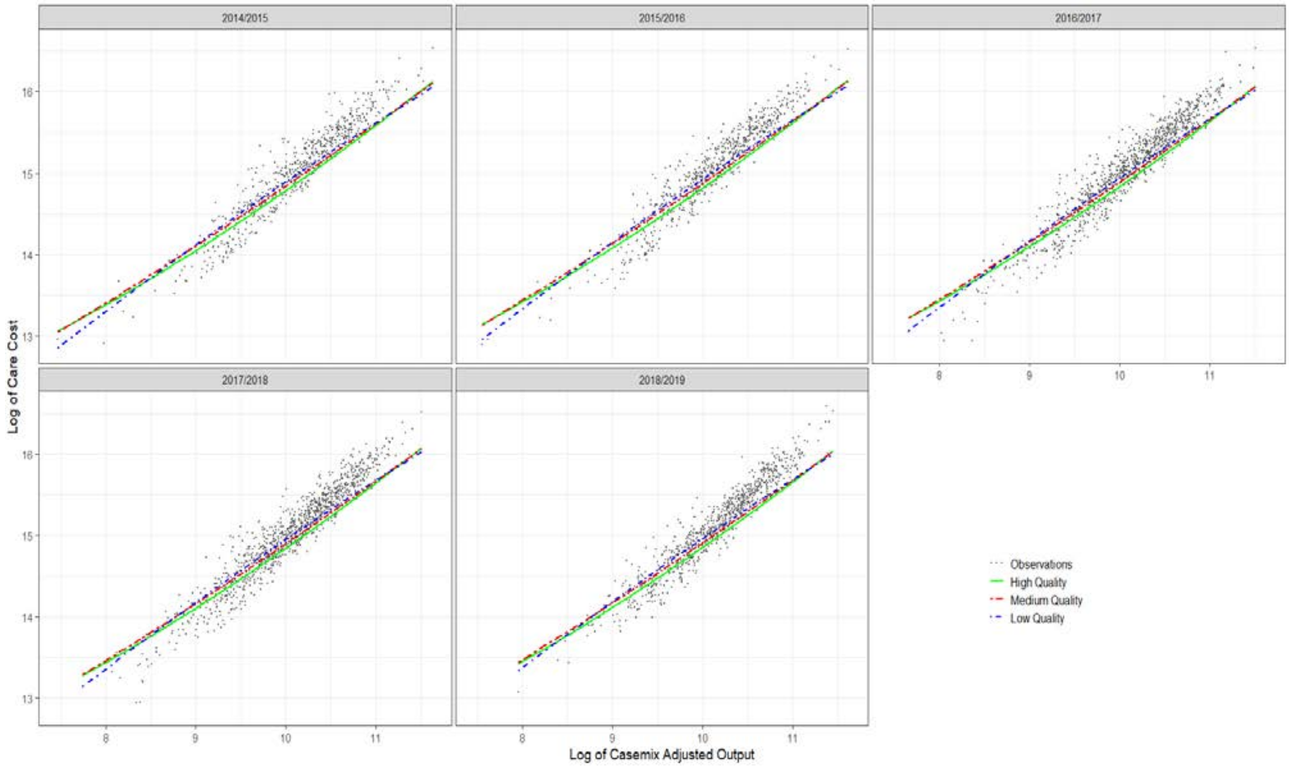

Figure 39. Estimated frontier for care cost across financial years - ALS77 model with flexible functional form (other cost categories fixed at their mean)

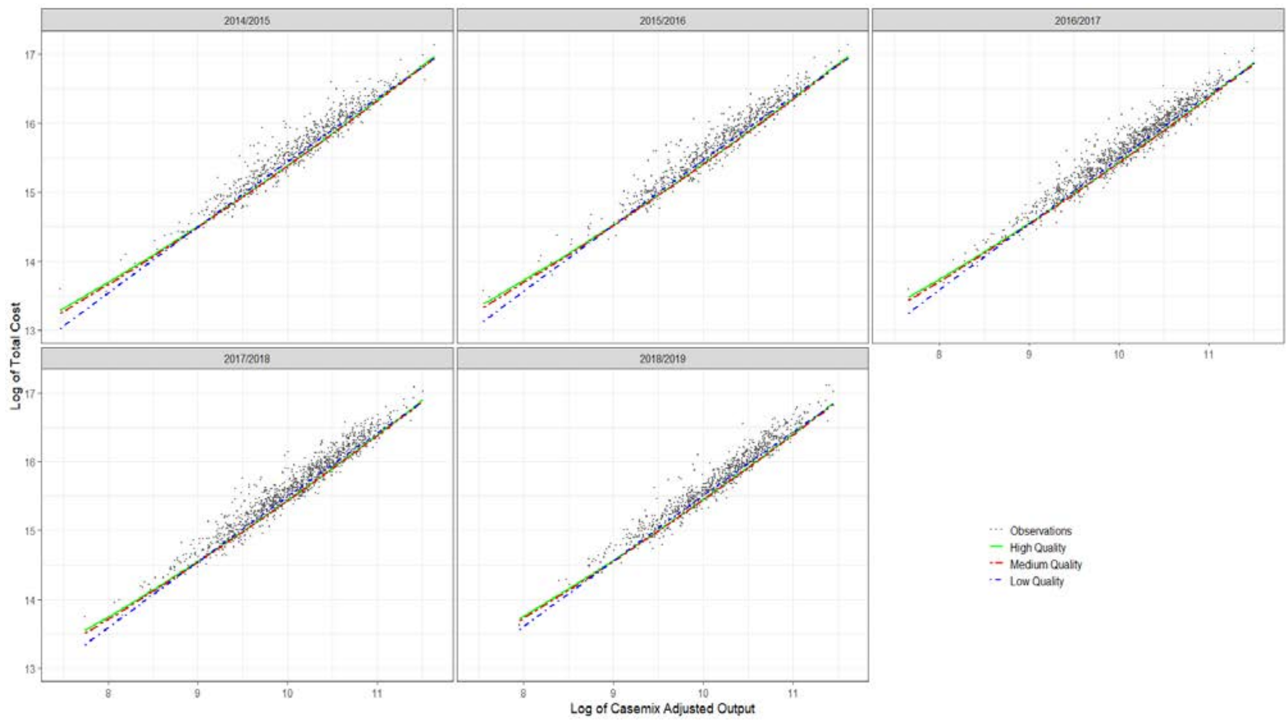

Figure 40. Estimated frontier for total cost across financial years - ALS77 model with flexible functional form

Table 24. Estimated inefficiency for care cost and total cost

	Model	Observations	Mean	Standard deviation	Minimum	Maximum	Histogram
Efficiency scores for care cost	ALS77	4,228	91.60%	2.70%	74.45%	96.80%	Figure 41
	CFG95	4,228	91.15%	4.84%	49.85%	97.01%	Figure 42
Efficiency scores for total cost	ALS77	4,228	88.39%	6.24%	53.72%	97.66%	Figure 43
	CFG95	4,228	88.81%	6.21%	52.17%	97.84%	Figure 44

ALS77: Aigner, Lovell and Schmidt (1977). CFG95: Caudill, Ford, and Gropper (1995). See Section 4.1.

Table 25. Estimated aggregate efficiency for care cost and total cost[35]

	Model	Aggregate Efficiency	Standard Error
Aggregate efficiency for care cost	ALS77	91.36%	0.04%
	CFG95	91.40%	0.06%
Aggregate efficiency for total cost	ALS77	87.73%	0.11%
	CFG95	88.28%	0.11%

ALS77: Aigner, Lovell and Schmidt (1977). CFG95: Caudill, Ford, and Gropper (1995). See Section 4.1.

Table 26. Marginal effects of explanatory variables on mean inefficiency for care cost and total cost

Variable	CFG95 for care cost Marginal effects	CFG95 for total cost Marginal effects	Impacts on inefficiency	Intuitive interpretation of the coefficients (on average and *ceteris paribus*) and explanations
Occupancy rate	-0.0016	-0.0018	Reduce	Higher occupancy rate is associated with higher cost efficiency, and less waste. Specifically, on average and ceteris paribus, when the occupancy rate increases by 1%, the mean inefficiencies in care cost and total cost are expected to decrease by 0.16% and 0.18%, respectively.
For-profit	-0.0260	-0.0582	Reduce	Compared to not-for-profit facilities, for-profit facilities appeared to be run with higher cost efficiency (i.e. used the budget more efficiently). **However, as noted earlier, this result needs to be interpreted with great caution as differences in quality achieved by the different ownership types could reflect a lack of differentiation within the three quality levels by the composite quality index.** To the extent that one ownership type consistently spends less and produces less quality, but the composite index is unable to distinguish those differences in quality, the efficiency analysis would incorrectly suggest the lower spending is 'efficient'.
Government	0.0988	0.0297	Increase	Government facilities were run with more cost inefficiency compared to not-for-profit facilities. The association is stronger for direct care cost than total cost. In addition to the caution noted above about the differences between ownership types may reflect unmeasured quality differences, we also note these facilities have more "spare capacity" in terms of employees, which can show up as inefficiency. Knowing that government facilities accounted for a large proportion of Q1 facilities, it is likely that this spare capacity is associated with high quality of care (see Q1 below).
Remote	0.0692	0.0626	Increase	Remote facilities are associated with higher cost inefficiency. Specifically, on average and ceteris paribus, the mean inefficiencies in care cost and total cost of remote facilities are 6.92% and 6.26% higher than those of non-remote facilities, respectively. This could be due to having

[35] See Simar and Zelenyuk (2018) for more details on formulas for the weighted mean and the variable. It is worth noting here that we use cost-analogue of the formulas and $E[exp(-u_i)|\varepsilon_i]$ are used in place of true efficiency scores.

				difficulty in getting resources, due to higher prices associated with resources, but also the extra need to have spare capacity.
Small	0.0406	-0.0316	Increase in total cost Reduce in care cost	Small-sized facilities are associated with higher inefficiency in care costs, but lower inefficiency in total cost. Specifically, on average and ceteris paribus, the mean inefficiency in care cost of small facilities is 6.26% higher than, but in total cost is 3.16% lower than, those of non-small facilities. Smaller facilities may be more efficient to run, which may be due to their smaller management structures and associated lower managerial labour costs.
Q1	0.0698	0.0259	Increase	Higher quality facilities are associated with higher cost inefficiency. On average and ceteris paribus, the mean inefficiencies in care cost and total cost of Q1 facilities are 6.98% and 2.59% higher than those of Q3 facilities, respectively. In order to maintain high quality of care, facilities might need allow more "spare capacity" in terms of employees, which can appear as inefficiency.
Q2	0.0110	0.0123	Not statistically significant	There is no significant statistical evidence about the difference in mean efficiency between Q2 and Q3 facilities.

CFG95: Caudill, Ford, and Gropper (1995). See Section 4.1. OBD: occupied bed day.

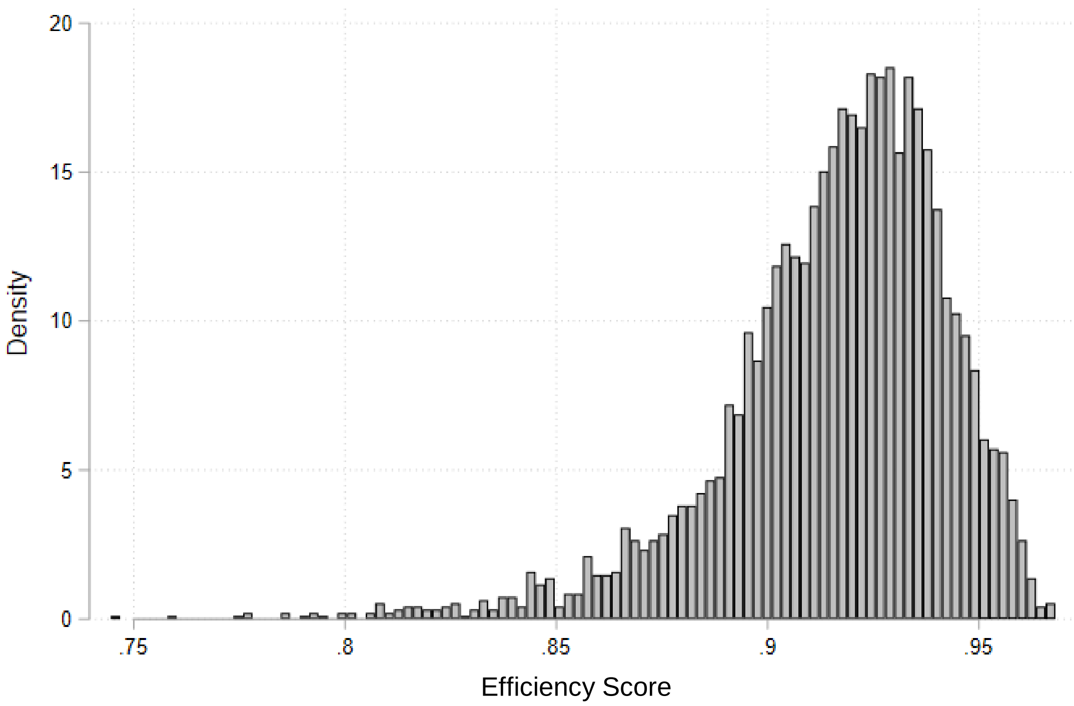

Figure 41. Histogram of estimated efficiency for care cost, ALS77 model

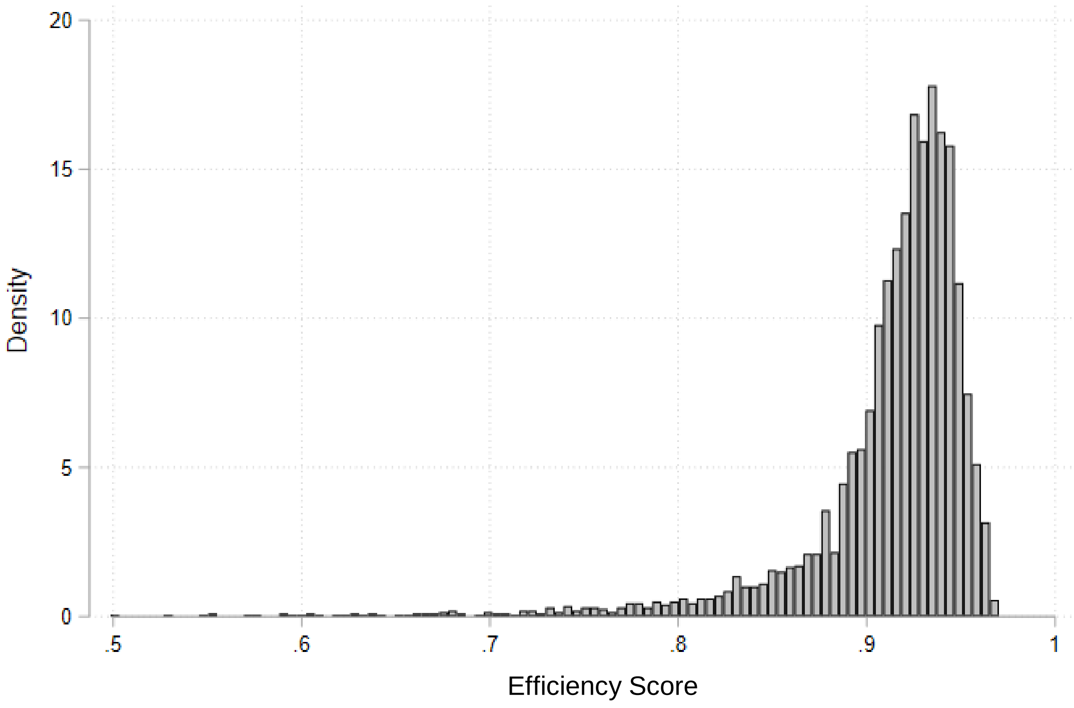

Figure 42. Histogram of estimated efficiency for care cost, CFG95 model

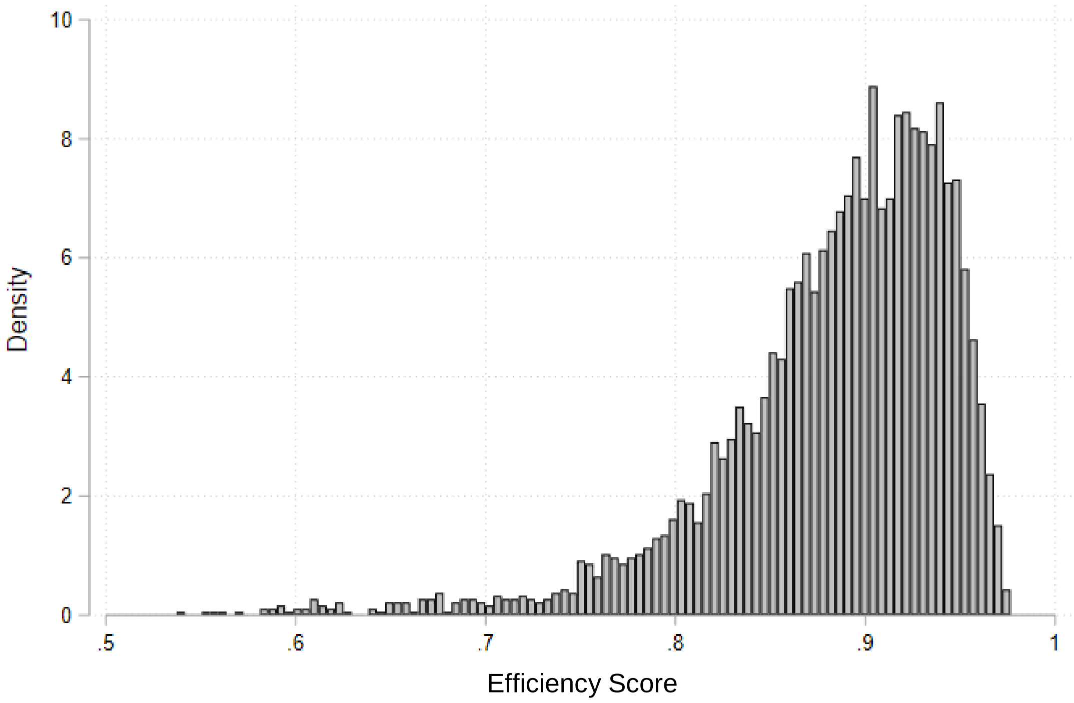

Figure 43. Histogram of estimated efficiency for total cost, ALS77 model

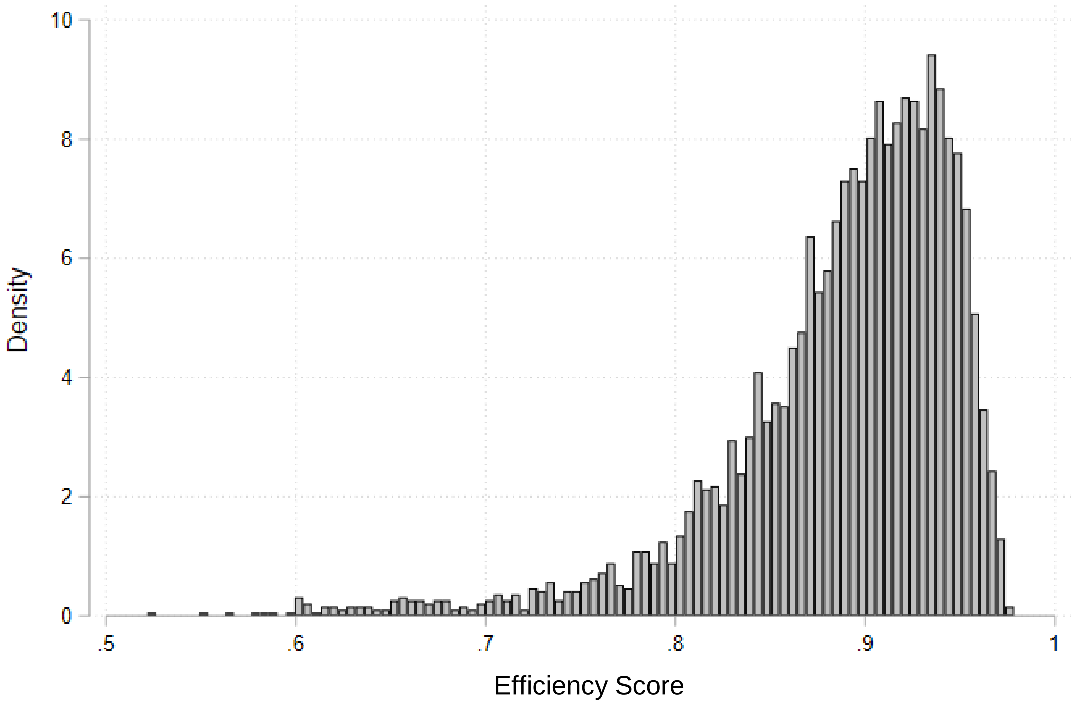

Figure 44. Histogram of estimated efficiency for total cost of care, CFG95 model

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